Digest 4, Homework 4

(A compilation of emailed homework questions, answered around Wednesday. Will possibly be updated late Wednesday night.)

Question. [Exercise 1] Let $G(u,v) = (3u+v, u-2v)$. Using the Jacobian, determine the area of $G(D)$ where D is defined as

- $D = [0,3] \times [0,5]$.
- $D = [2,5] \times [1,7]$.

(From a student:) I would like to know whether the domain is in terms of x and y or in terms of u and v. Also, I can't tell whether I'm supposed to use G to change u and v into x and y or the other way around.

Answer. The variables x and y do not appear in the problem. If you would like, I can write $D = [0,3] \times [0,5]$, so that D is the set of ordered pairs $\{(u,v) \colon 0 \leq u \leq 3 \text{ and } 0 \leq v \leq 5\}$. We then define $G(D)$ to be the set of ordered pairs $\{G(u,v) \colon 0 \leq u \leq 3 \text{ and } 0 \leq v \leq 5\}$. That is, $G(D)$ is the set of ordered pairs $\{(3u+v, u-2v) \colon 0 \leq u \leq 3 \text{ and } 0 \leq v \leq 5\}$. So, I believe $G(D)$ looks like some kind of parallelogram. Perhaps part of the confusion is that in class I always wrote x and y for the variables, and I used u and v for functions, whereas here I am using u and v for variables? In such a case, just note that the variable names do not matter (just as they don't matter for one-variable substitutions), and so you should be able to do the problem with any choice of variable names.

Question. [Exercise 4] Find the center of mass of the region in the plane that is bounded by the positive x axis, the positive y axis, and the graph of the function $y = e^{-x}$.

(From a student:) Should I assume the region has constant density 1?

Answer. Yes.

Question. [Exercise 8] Let D be the region in the plane where $x > 0$, $y > 0$, and which is bounded by the curves $xy = 1$, $xy = 9$, $y = x$ and $y = 4x$. Using the transformation $x = u/v$ and $y = uv$ with $u > 0$ and $v > 0$, evaluate the following integral.

$$\iint_D \left(\sqrt{\frac{y}{x}} + \sqrt{xy} \right) dxdy.$$

(Hint: use $G(x,y) = (\sqrt{xy}, \sqrt{y/x}) = (u(x,y), v(x,y))$.)

(From a student:) Which function f should I use in the change of variables formula?

Answer. I think you want to use the function $f(u,v) = (u/v)(u+v) = u + u^2/v$. This function f is chosen to "cancel" the Jacobian term that comes from the change of variables formula.

Question. [Exercise 9] Let D denote the plane $z = 0$ in Euclidean space $\mathbf{R}^3$. Let S denote the unit sphere $x^2+y^2+z^2 = 1$. Consider the function $G\colon D \to S$ where $G(x,y,0)$ is defined to be the point of intersection of S with the straight line that goes through $(x,y,0)$ and $(0,0,1)$.

- Find a formula for G in terms of x and y. That is, find functions $u(x,y), v(x,y)$ and $w(x,y)$ such that $G(x,y,0) = (u(x,y), v(x,y), w(x,y))$. (Hint: First draw a picture. Let L denote the line that goes through ....)
- Is G a one-to-one correspondence?

...

(From a student:) Do I need to use the quadratic equation to solve for c?

Answer. You should be able to solve for c without using the quadratic equation. You could use the quadratic equation if you want, but there is a simpler way. More specifically, you should arrive at the equation $x^2(1-c)^2 + y^2(1-c)^2 = 1 - c^2$. Now, divide both sides by $(1-c)$.

Question. I would like to know how to study for the future exam. I studied over the lecture notes, textbook, homework and test resources you gave us. However, I found out that the exam was kind of surprising that I did not see such question types before. And also I realized that the textbook did not help me that much since question 4 was much more complicated than book problems. Could you suggest me a way of preparing for the test? I would not mind for this first exam since one can be dropped but I want to do much better on the second midterm.

Answer. Some of the questions (for example, question 4) were designed to be challenging, and they were designed to be different than any of the homework problems. In particular, I think only two or three people got question 4 entirely correct (this was not my intention but it happened). There are many facets to your questions, so I would like to address them separately. First, how should you prepare for exams? I provided several exam resources that I considered to be of comparable difficulty to our midterm. For example, the "bonus" questions from the practice exam were presumably comparable to question 4 on our exam in terms of difficulty. However, I would agree that perhaps textbook problems might give you the impression that a lot of problems might not be so hard. I would therefore recommend focusing on the difficult problems in the textbook, since they should test the limits of your understanding better than the easier exercises. And then also focus on some nice practice exam questions. Generally, if there is a problem you are having trouble with, then there is something more for you to learn. From this same theme, I will quote Professor Tao's blog:

"Its also best to keep in mind that obtaining a solution is only the short-term goal of solving a mathematical problem. The long-term goal is to increase your understanding of a subject. A good rule of thumb is that if you cannot adequately explain the solution of a problem

to a classmate, then you haven't really understood the solution yourself, and you may need to think about the problem more (for instance, by covering up the solution and trying it again)."

Your question about exam studying also seems to suggest that a day or two of studying can alter your outcome on an exam. I disagree with this idea. Your performance on the exam results from your studying throughout the entire course, coming to office hours as needed, etc. For example, when I was an undergraduate, I started studying for an exam one week before it started, then I studied most intensely two days before the exam. And a day before the exam I did not study at all, to relax and regain some energy. On the day of the exam I just did some light reviewing but nothing strenuous. This strategy worked well for me and maybe it can work well for you.

Lastly, if your performance improves throughout the quarter, your final letter grade will be higher than you might expect. That is, improvement over the quarter is rewarded.