Digest 9

(A compilation of emailed homework questions, answered around Wednesday.)

Question. [Exercise 2] Let $f\colon \mathbf{R}^2 \to \mathbf{R}$. Polar coordinates r, θ are defined by $x = r\cos\theta$ and $y = r\sin\theta$. Rewrite f_x just in terms of derivatives of f with respect to r and with respect to θ. Also, the expression for f_x should only be in terms of r and θ.

(From a student): Are we supposed to rewrite the function given in #1 ($f(x, y, z) = x^2+2yz$) in terms of r and θ or is there some other function I'm not noticing?

Answer. You are supposed to consider an arbitrary function f in Exercise 2. You should not use any specific function.

Question. [Exercise 2] Let $f\colon \mathbf{R}^2 \to \mathbf{R}$. Polar coordinates r, θ are defined by $x = r\cos\theta$ and $y = r\sin\theta$. Rewrite f_x just in terms of derivatives of f with respect to r and with respect to θ. Also, the expression for f_x should only be in terms of r and θ.

(From a student): Are you asking us to find $\partial f/\partial r$, and $\partial f/\partial\theta$? or are you asking us to find $\partial f/\partial x$ in terms of r and θ?

Answer. You should find $\partial f/\partial x$ in terms of r and θ.

Question. [Exercise 4] Suppose $x^2 + 2yz + z^3 - 5 = 0$. Compute $\partial y/\partial z$.

(From a student): When we compute $\partial y/\partial z$ for the equation, wouldn't we get 0 since y is not a function of z? or it is, so that we use the pattern of the implicit differentiation?

Answer. For the purpose of this problem, we think of y as an implicit function of z, so that implicit differentiation is what you want to do.

Question. [Exercise 5] Let $f(x, y) = x^2 + e^x y + \sin(xy)$. Identify the critical points of f, and identify these points as local maxima, local minima or saddle points.

(From a student): I was looking for when the gradient of exercise 5 in the assignment nine is equal to zero in order to find the critical points and I couldn't find it is there any problem? Or should I just keep looking?

Answer. Hm, looks like that question was unintentionally hard; I will replace it with an easier problem.

Question. [Exercise 6] Find the maximum and minimum values of the function $f(x, y) = x^2 + yx$ on the disk $x^2 + y^2 \leq 1$.

(From a student): When I substitute $y = (1 - x^2)^{1/2}$, and find the derivative of the new function, I want to set the equation $2x(1 - x^2)^{1/2} + 1 - x^2 - x^2$ to 0. But it turns out this is a equation that has super complex results. Could you please tell me should I use another way to do this problem or I should use calculator?

Answer. You should be able to do this without using a calculator. Also you may want to consider the negative root as well.

Question. (From a student): Could you provide more practice exams?

Answer. Sure, in addition to the ones already listed on the course websites, here are some more: (ignore any questions about geometric series, probability, or about integration of multivariable functions):

Near the bottom of this page are several practice finals:
http://math.ucsd.edu/ justin/10CW15/10CW15.html

A midterm-length exam

Solutions

Question. (From a student): When we compute $D_v f(a, b)$ where v is not a unit vector, do we have to convert it into a unit vector or simply compute as $\nabla f(a, b) \cdot v$?

Answer. You should use the second method. As defined in class, $D_v f(a, b) = \nabla f(a, b) \cdot v$. The book only considers $D_v f(a, b)$ when v is a unit vector, but we can see that this definition still makes sense for any vector v.

Question. (From a student): Could you provide a hint for Exercise 13 on the homework?

Answer. One way of doing the problem is to maximize $f(r, b) = (1/3)\pi r^2 b$ subject to the constraint $g(r, b) = \pi r\sqrt{r^2 + b^2} = c$. However, this way of using Lagrange multipliers can be pretty difficult. Instead, it is easier to maximize $f(r, b) = (1/3)\pi r^2 b$ subject to the constraint $g(r, b) = \pi^2 r^2 (r^2 + b^2) = c^2$, which is a much more tractable (and equivalent) Lagrange multipliers problem.

Question. (From a student): On homework 8, we were asked to evaluate $D_v f$. I thought this meant we were supposed to normalize the vector v, since that was how the book did it. But, as I saw in the solutions, this was not the case. Im just confused about when and why I should normalize v.

Answer. When $f : \mathbf{R}^2 \to \mathbf{R}$ and $v \in \mathbf{R}^2$, the book defines $D_v f(a, b) = \nabla f(a, b) \cdot v$. We made the same definition in class. The book only considers this definition when v is a unit vector. But the definition works equally well when v is not a unit vector. So, if I ever ask you to compute $D_v f(a, b)$, I just expect you to plug into this formula $D_v f(a, b) = \nabla f(a, b) \cdot v$ for whatever vector v is given (whether or not it is a unit vector).