Please provide complete and well-written solutions to the following exercises.

No due date, but the quiz in Week 5 in the discussion section (on October 27th or 29th) will be based upon this homework.

Assignment 4

Exercise 1. Suppose $y(x)$ satisfies $\sqrt{x+y}=1+x^2y^2$. Find dy/dx by implicit differentiation.

Exercise 2. One of the formulas for inventory management says that the average weekly cost of ordering, paying for, and holding merchandise is

$$A(q)=\frac{km}{q}+cm+\frac{hq}{2},$$

where q is the quantity you order when things run low (shoes, radios, brooms, or whatever the item might be); k is the cost of placing an order (the cost is the same, no matter how often you make an order); c is the cost of one item (a constant); m is the number of items sold each week (a constant); and h is the weekly holding cost per item (a constant that takes into account things such as space, utilities, insurance, and security). Find dA/dq and d^2A/dq^2, and interpret your results in terms of the constants.

Exercise 3. Find the equation of the tangent to the curve at the given point.

(a) $y=4(\sin(x))^2$, $(x,y)=(\pi/6,1)$, (b) $y=\frac{x^2-1}{x^2+1}$, $(x,y)=(0,-1)$.

Exercise 4. Find the equation of the tangent line to the curve at the given point.

(a) $y=\sqrt{1+4\sin(x)}$, $(x,y)=(0,1)$, (b) $x^2+4xy+y^2=13$, $(x,y)=(2,1)$.

Exercise 5. In the late 1860s, A. Fick, a professor of physiology in the Faculty of Medicine in Würzberg, Germany, developed one of the methods we use today for measuring how much blood your heart pumps in a minute. Your cardiac output as you read this sentence is probably about 7 L/min. At rest it is likely to be a bit under 6 L/min. If you are a trained marathon runner running a marathon, your cardiac output can be as high as 30 L/min.

Your cardiac output can be calculated with the formula

$$y=\frac{Q}{D},$$

where Q is the number of mLs of CO_2 you exhale in a minute, and D is the difference between the CO_2 concentration (mL/L) in the blood pumped to the lungs and the CO_2 concentration in the blood returning from the lungs. With $Q=233$ mL/min and $D=97-56=41$ mL/L,

$$y=\frac{233\text{mL/min}}{41\text{mL/L}}\approx 5.68\text{L/min},$$

fairly close to the 6 L/min that most people have at basal (resting) conditions.

Suppose that when $Q = 233$ and $D = 41$, we also know that D is decreasing at the rate of 2 units a minute but that Q remains unchanged. What is happening to the cardiac output?

Exercise 6. A lighthouse sits 1 mile from the shore. Let P be the point on the shore that is closest to the lighthouse. The light completes four revolutions per minute, at a constant speed. How fast is the light moving along the shore, when the light is 1 mile above the point P?

Exercise 7. Coffee is draining from a conical filter into a cylindrical coffeepot at a rate of 10 in^3/min. The filter is a circular cone with a height of 6 inches and a 6 inch diameter. The cylindrical coffeepot has a diameter of 6 inches.

(a) How fast is the level in the pot rising when the coffee in the cone is 5 inches deep?
(b) How fast is the level in the cone falling at this point in time?

Exercise 8. A 20 foot lamppost is casting a shadow on a person who is 6 feet tall. At time $t = 0$ (where t is measured in seconds), the person is standing 10 feet from the lamppost, and she then proceeds walking away from the lamppost at a rate of 3 feet per second. What is the rate of change of the length of her shadow at time $t = 2$? (That is, how quickly is the shadow lengthening, with respect to time?)