Analysis 1, Fall 2014, UCLA — Instructor: Steven Heilman

Name: ____________________ UCLA ID: ______________ Date: ____________

Signature: ____________________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 1

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right. Good luck!

1. (10 points) Prove the following statement.

   Let n be a positive integer. Then $1 + 2 + \cdots + n = n(n+1)/2$.

2. (a) (4 points) Define a Cauchy sequence of rational numbers.

(b) (6 points) Let $(a_n)_{n=0}^{\infty}$ and let $(b_n)_{n=0}^{\infty}$ be Cauchy sequences of rational numbers. Prove that $(a_n + b_n)_{n=0}^{\infty}$ is a Cauchy sequence of rational numbers.

3. Determine which of the following sequences converges. If the sequence converges, find its limit and prove that the sequence converges to this limit. If the sequence does not converge, prove it.

   (a) (5 points) Let $a_n = 1/n$ for any positive integer n.

   (b) (5 points) Let $a_n = (-1)^n$ for any positive integer n.

4. (10 points) Let X and Y be countable, disjoint sets. Prove that $X \cup Y$ is a countable set. (Recall that a set A is countable if and only if, there exists a bijection $f\colon A \to \mathbf{N}$. That is, for every $n \in \mathbf{N}$ there exists exactly one $a \in A$ such that $f(a) = n$.)

5. (10 points) The following inductive argument is incorrect. Explain the flaw in the argument.

   Claim: All horses on Earth are the same color.

   Proof: We prove the claim by induction. Let k be a positive integer. In the case $k = 1$, a single horse has the same color as itself, so the case $k = 1$ of the induction is known. We now assume by induction that each set of k horses is of the same color. We want to show that a set of $k + 1$ horses is of the same color. Suppose I have a set C of $k + 1$ horses. If I remove one horse from this set of $k + 1$ horses, I have k horses of the same color, by the inductive hypothesis. Label this set of k horses as A. All horses in the set A are the same color.

   Now, take the set of $k + 1$ horses and remove a different horse from this set than the one that we removed before. Label this new set of k horses as B. All horses in the set B are the same color, by the inductive hypothesis. Since A and B have some horses in common, the $(k + 1)$ horses in C all must have the same color. We have therefore completed the induction, and the claim is proven. □

(Scratch paper)