Please provide complete and well-written solutions to the following exercises.

Due May 1, 1159PM PST, to be uploaded as a single PDF document to Brightspace.

# Homework 7

**Exercise 1.** Show that, in a Condorcet election with three candidates, if we use a majority vote to compare each pair of candidates, then as the number of voters goes to infinity, the probability that a Condorcet winner occurs approaches

$$\frac{3}{4} + \frac{3}{2\pi} \sin^{-1}(1/3).$$

(Assume that each voter ranks their candidates uniformly at random and independently of all other voters.)

**Exercise 2.** Create an algorithm that finds an evolutionarily stable strategy in a two-person general sum game (if the strategy exists). Find a reasonable bound on the run time of this algorithm.

**Exercise 3.** Find all Nash equilibria of a two-player symmetric game with the following payoffs

$$\begin{pmatrix} 4 & 3 & 2 & 5 & 6 \\ 3 & 1 & 8 & 9 & 1 \\ 7 & 0 & 7 & 0 & 7 \\ 1 & 3 & 3 & 2 & 1 \\ 0 & 1 & 2 & 9 & 1 \end{pmatrix}$$

(Hint: maybe you should use a computer program.)

Find all evolutionary stable strategies.

(Hint: maybe you should use a computer program.)

**THE EXERCISES BELOW ARE OPTIONAL. You do not have to complete them. They are just for your benefit for studying for the final.**

**Exercise 4** (Optional)**.** Prove a Hoeffding inequality for random variables $X_1, \ldots, X_n$ that are 1-sub-Gaussian.

A real-valued random variable $X$ is called $k$-sub-Gaussian if

$$\mathbf{E} e^{[X - \mathbf{E}X]t} \leq e^{k^2 t^2/2}, \qquad \forall\, t \in \mathbf{R}.$$

**Exercise 5** (Optional)**.** Write a computer program that implements the bandit algorithms we discussed in class. (Explore-then-Commit, Successive Elimination, and UCB) Consider e.g. some rewards with different expected values, and plot the regret over time. Compare your findings with our theoretical regret bounds.

Try also rewards that are Gaussian random variables with different means. How do your regret bounds behave?

Try also rewards that are Poisson with different means. How do your regret bounds behave?