Please provide complete and well-written solutions to the following exercises.

Due January 23 1159PM PST, to be uploaded as a single PDF document to Brightspace.

Homework 1

Exercise 1. As needed, refresh your knowledge of proofs and logic by reading the following document by Michael Hutchings: http://math.berkeley.edu/∼hutching/teach/proofs.pdf

Exercise 2. Read the game rules for connect four, checkers, chess and go. We will be discussing these examples in class throughout the quarter.

Even if you are familiar with chess already, make sure to read the entire set of rules. There are many rules about stalemates that are a bit obscure, but they are important for this class, since these rules force the game to not last forever.

Exercise 3. Take the following quizzes on logic, set theory, and functions. (This material should be review from 115A.):

http://scherk.pbworks.com/w/page/14864234/Quiz%3A%20Logic
http://scherk.pbworks.com/w/page/14864241/Quiz%3A%20Sets
http://scherk.pbworks.com/w/page/14864227/Quiz%3A%20Functions

(These quizzes are just for your own benefit; you don't need to record your answers anywhere.)

Exercise 4. Prove the following assertion by induction:

For any natural number n, $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$.

Exercise 5. Write the following numbers in binary: $1, 3, 5, 8, 9$. Compute the following sums modulo 2: $1 + 3$, $4 + 9$, $10^{10} + 1$.

Exercise 6. Tic-tac-toe is a partisan combinatorial game. Prove that it is progressively bounded. Then, try to figure out which of the following situations is true: the first player has a winning strategy; the second player has a winning strategy; both players have a strategy forcing at least a draw. (You do not need to give a formal proof of which situation holds, just try to guess the answer by playing a few tic-tac-toe games and by drawing on your personal experience.)

Exercise 7. Let n be a positive integer. Consider the game Chomp played on an $n \times n$ board. Explicitly describe the winning strategy for the first player. (Hint: the first move should remove the square which is diagonally adjacent to the lower left corner.)

Exercise 8. Consider the game of Chomp played on a board of size $2 \times \infty$. Recall that a typical Chomp game board is $n \times m$, so that the board has n rows and m columns. We can label the rows as $\{1, 2, \ldots, n\}$ and we can label the columns as $\{1, 2, \ldots, m\}$, where n, m are positive integers. On a $2 \times \infty$ board, we label the rows as $\{1, 2\}$, and we label the columns as $\{1, 2, 3, 4, 5, 6, \ldots\}$. We can think of the row and column labels as coordinates in the xy-plane. So, the lower left corner will still have x-coordinate 1 and y-coordinate 1, so that the lower left square has coordinates $(1, 1)$; the square to the right of this has coordinates $(2, 1)$, and so on.

On the $2 \times \infty$ board, which player has a winning strategy? Prove your assertion, and describe explicitly the winning strategy.

Let $n > 2$ be an integer. On the $n \times \infty$ board, which player has a winning strategy? Prove your assertion, and describe explicitly the winning strategy.

On the $\infty \times \infty$ board, which player has a winning strategy? Prove your assertion, and describe explicitly the winning strategy.