Math 458, Fall 2022, USC   Instructor: Steven Heilman

Name: _____   USC ID: _____   Date: _____

Signature: _____. Discussion Section: _____
(By signing here, I certify that I have taken this test while refraining from cheating.)

Exam 1

This exam contains 8 pages (including this cover page) and 5 problems. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.

- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right. Good luck![a]

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1. Label the following statements as TRUE or FALSE. If the statement is true, **explain your reasoning**. If the statement is false, **provide a counterexample or explain your reasoning**.

   (a) (2 points) The smallest positive number that exists in double precision floating point arithmetic is
   $$2^{-1022}.$$
   TRUE   FALSE   (circle one)

   (b) (2 points) If we enter the following command into the Matlab command line
   ```
   1:100
   ```
   and then press the 'Enter' key, Matlab produces a row vector of integers $(1, 2, 3, \ldots, 99, 100)$.
   TRUE   FALSE   (circle one)

   (c) (2 points) When entered into Matlab, the following commands
   ```
   for i=1:100
       i;
   end
   ```
   will print out the consecutive integers $1, 2, 3, \ldots, 99, 100$.
   TRUE   FALSE   (circle one)

(d) (2 points) If we enter the following command into the Matlab command line
$$(2^{\wedge}53) * (1- (1+2^{\wedge}(-53)))$$
and then press the 'Enter' key, Matlab outputs 1.

TRUE   FALSE   (circle one)

(e) (2 points) If we enter the following command into the Matlab command line
$$((2<4) \ \& \ (4<3)) \ | \ {\sim}(2<7)$$
and then press the 'Enter' key, Matlab outputs 1.

TRUE   FALSE   (circle one)

2. (10 points) Solve the linear system of equations

$$Ax = b$$

where $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbf{R}^3$ is unknown, $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

3. (10 points) Newton's Method tries to find a zero of a function $f : \mathbf{R} \to \mathbf{R}$ with initial guess $x_0 \in \mathbf{R}$. The iteration for Newton's Method is

$$x_{n+1} := x_n - \frac{f(x_n)}{f'(x_n)}, \qquad \forall\, n \geq 0.$$

- Write a program in Matlab that starts with an initial guess $x_0 = 5$, that outputs the 10th Newton's method iteration x_{10}, when

$$f(x) = x^2 - 5, \qquad \forall\, x \in \mathbf{R}.$$

- Describe a real number $t \in \mathbf{R}$ that will satisfy $|t - x_{10}| < 2^{-48}$. Justify your answer. (In your justification, just for this question, you are allowed to use any result from the homework.)

4. (10 points) Let A be an $n \times n$ real matrix. Let $\det(A)$ denote the determinant of A.

   Describe in detail an algorithm for computing $|\det(A)|$ that uses at most $100n^5$ arithmetic operations.

   Make sure to justify the number of arithmetic operations that you use.

   (Hint: use the LU decomposition.)

5. (10 points) Give an example of a 2×2 real matrix A such that A **cannot** be written as $A = LU$ where L is a lower triangular 2×2 real matrix with ones on its diagonal, and U is an upper triangular 2×2 real matrix.

You must fully justify your answer.

(Scratch paper)