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Please provide complete and well-written solutions to the following exercises.

Due October 5, in the discussion section.

(This Review Assignment will be collected, but this Review Assignment will not be graded.)

## Preliminary Review Assignment

**Exercise 1.** As needed, refresh your knowledge of proofs and logic by reading the following document by Michael Hutchings: <http://math.berkeley.edu/~hutching/teach/proofs.pdf>

**Exercise 2.** Take the following quizzes on logic, set theory, and functions:

<http://scherk.pbworks.com/w/page/14864234/Quiz%3A%20Logic>

<http://scherk.pbworks.com/w/page/14864241/Quiz%3A%20Sets>

<http://scherk.pbworks.com/w/page/14864227/Quiz%3A%20Functions>

(These quizzes are just for your own benefit; you don't need to record your answers anywhere.)

**Exercise 3.** Prove the following assertion by induction:

For any natural number  $n$ ,  $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$ .

**Exercise 4.** Prove that the set of real numbers  $\mathbf{R}$  can be written as the countable union

$$\mathbf{R} = \bigcup_{j=1}^{\infty} [-j, j].$$

(Hint: you should show that the left side contains the right side, and also show that the right side contains the left side.)

Prove that the singleton set  $\{0\}$  can be written as

$$\{0\} = \bigcap_{j=1}^{\infty} [-1/j, 1/j].$$

**Exercise 5** (Continuity of a Probability Law). Let  $\mathbf{P}$  be a probability law on a sample space  $\Omega$ . Let  $A_1, A_2, \dots$  be sets in  $\Omega$  which are increasing, so that  $A_1 \subseteq A_2 \subseteq \cdots$ . Then

$$\lim_{n \rightarrow \infty} \mathbf{P}(A_n) = \mathbf{P}(\cup_{n=1}^{\infty} A_n).$$

In particular, the limit on the left exists. Similarly, let  $A_1, A_2, \dots$  be sets in  $\Omega$  which are decreasing, so that  $A_1 \supseteq A_2 \supseteq \cdots$ . Then

$$\lim_{n \rightarrow \infty} \mathbf{P}(A_n) = \mathbf{P}(\cap_{n=1}^{\infty} A_n).$$

**Exercise 6.** Retake at least one of the finals I gave when I taught math 170A:

<http://www.math.ucla.edu/heilman/teach/170afinal.pdf>

<http://www.math.ucla.edu/heilman/teach/170afinalsoln.pdf>

<http://www.math.ucla.edu/heilman/teach/170afinalv2.pdf>

<http://www.math.ucla.edu/heilman/teach/170afinalv2soln.pdf>