Probability, 170A, Spring 2017, UCLA | Instructor: Steven Heilman

Name: ______________________ UCLA ID: ______________ Date: ___________

Signature: ______________________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 2

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam. You are required to show your work on each problem on the exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a theorem or proposition from class or the notes or the book you must indicate this** and explain why the theorem may be applied. It is okay to just say, "by some theorem/proposition from class."
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right. Good luck![a]

1. Label the following statements as TRUE or FALSE. If the statement is true, **explain your reasoning**. If the statement is false, **provide a counterexample and explain your reasoning**.

   (a) (2 points) The number of permutations of the set $\{1, 2, \ldots, n\}$ is $n!$.

   TRUE FALSE (circle one)

   (b) (2 points) There is some random variable X such that $\text{var}(X) = -1$.

   TRUE FALSE (circle one)

   (c) (3 points) (h) Let X be a random variable that only takes nonnegative integer values. Assume that for any integer $n > 10$, we have $\mathbf{P}(X \geq n) = 1/\sqrt{n}$. Then $\mathbf{E}(X) < \infty$.

   TRUE FALSE (circle one)

   (d) (3 points) Let X, Y be discrete random variables. Assume that $\mathbf{E}(X|Y = 1) = \mathbf{E}(X|Y = 2) = 2$, and for any integer $k \geq 3$, assume that $\mathbf{E}(X|Y = k) = 0$. Also, assume that $\mathbf{P}(Y = k) = 2^{-k}$ for any integer $k \geq 1$. Then $\mathbf{E}(X) = 2$.

   TRUE FALSE (circle one)

2. (10 points) Let X, Y be random variables with joint PMF $p_{X,Y}$ such that

$$p_{X,Y}(x, y) = \mathbf{P}(X = x, Y = y) = 1/9, \quad \text{for all integers} \quad 1 \leq x, y \leq 3$$

Compute the probabilities of the following events.

- $X > 1$.
- $X + Y \leq 2$.
- $X^2 + Y^2 > 2$.

3. (10 points) Compute the mean and variance of a Poisson random variable X with parameter $\lambda = 1$. (Recall that $\mathbf{P}(X = k) = \frac{1}{e}\frac{1}{k!}$ for any nonnegative integer k.)

4. (10 points) Suppose you have \$100, and you need to come up with \$1000. You are a terrible gambler but you decide you need to gamble your money to get \$1000. For any amount of money M, if you bet \$$M$, then you win \$$M$ with probability .3, and you lose \$$M$ with probability .7. (If you run out of money, you stop gambling, and if you ever have at least \$1000, then you stop gambling.) Consider the following two possible strategies for gambling:

   Strategy 1. Bet as much money as you can, up to the amount of money that you need, each time.

   Strategy 2. Make a small bet of \$10 each time.

   Explain which strategy is better. That is, explain which strategy has a higher probability of getting \$1000.

5. (10 points) Suppose you have a standard 52-card deck of playing cards. (So the cards are sitting in a deck one on top of the other; there are thirteen cards of each of the four suits: hearts, spades, diamonds and clubs. And all permutations of the cards as a single deck of cards are equally likely.) Suppose you are drawing cards from the top of the deck without replacing them, and you put the cards in a pile. What is the expected number of cards you have to draw from the top of the deck before you find **two hearts**? (That is, what is the expected number of cards you have to draw out of the deck right before the pile goes from having one heart to having two hearts?)

(Scratch paper)