Probability, 170A, Spring 2017, UCLA — Instructor: Steven Heilman

Name: ______________________ UCLA ID: ______________ Date: ___________

Signature: ______________________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 1

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam. You are required to show your work on each problem on the exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a theorem or proposition from class or the notes or the book you must indicate this** and explain why the theorem may be applied. It is okay to just say, "by some theorem/proposition from class."
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Do not write in the table to the right. Good luck![a]

Problem	Points	Score
1	8	
2	10	
3	10	
4	10	
5	10	
Total:	48	

1. Label the following statements as TRUE or FALSE. If the statement is true, **explain your reasoning**. If the statement is false, **provide a counterexample and explain your reasoning**.

(a) (2 points) The negation of the statement
"There exists an integer j such that $j^4 - j < 5$" is:
"For every integer j, we have $j^4 - j \geq 5$."

TRUE FALSE (circle one)

(b) (2 points) Let $\mathbf{P}$ be the uniform probability law on $[0,1]$. Let $x_1, x_2, \ldots \in [0,1]$ be a countable set of distinct points. Then

$$\mathbf{P}\left(\cup_{n=1}^{\infty}\{x_n\}\right) = 0.$$

TRUE FALSE (circle one)

(c) (2 points) Let $\Omega = \{1,2,3,4,5,6,7\}$. For any $A \subseteq \Omega$, define $\mathbf{P}(A)$ to be the number of elements in A. Then $\mathbf{P}$ is a probability law on Ω.

TRUE FALSE (circle one)

(d) (2 points) Let $A_1, \ldots, A_n$ be disjoint events in a sample space Ω. That is, $A_i \cap A_j = \emptyset$ whenever $i, j \in \{1, \ldots, n\}$ satisfy $i \neq j$. Let $\mathbf{P}$ be a probability law on Ω. Assume $\mathbf{P}(A_i) > 0$ for all $1 \leq i \leq n$. Let $B \subseteq \Omega$. Then

$$\mathbf{P}(B) = \sum_{i=1}^{n} \mathbf{P}(B \cap A_i) = \sum_{i=1}^{n} \mathbf{P}(A_i)\mathbf{P}(B|A_i).$$

TRUE FALSE (circle one)

2. (10 points) Let $\Omega = [0, 1]$. For any $A \subseteq \Omega$, define $\mathbf{P}(A)$ so that

$$\mathbf{P}(A) := \begin{cases} 1 & , \text{if } \frac{1}{3} \in A \\ 0 & , \text{if } \frac{1}{3} \notin A. \end{cases}$$

Verify that $\mathbf{P}$ is a probability law on Ω. (When you verify Axiom (ii), you should consider countable unions of sets.)

3. (10 points) Suppose a test for a disease is 97% accurate. That is, if you have the disease, the test will be positive with 97% probability. And if you do not have the disease, the test will be negative with 97% probability. Suppose also the disease is fairly rare, so that roughly 1 in $1,000$ people have the disease. If you test positive for the disease, with what probability do you actually have the disease? (Hint: let B be the event that you test positive for the disease. Let A be the event that you actually have the disease. Compute a conditional probability.)

4. (10 points) An urn contains three red cubes and two blue cubes. A cube is removed from the urn uniformly at random. If the cube is red, it is kept out of the urn and a second cube is removed from the urn. If the cube is blue, then this cube is put back into the urn and an additional red cube is put into the urn, and then a second cube is removed from the urn.

   - What is the probability that the second cube removed from the urn is red?
   - If it is given information that the second cube removed from the urn is red, then what is the probability that the first cube removed from the urn is blue?

5. (10 points) Let A, B, C be independent events. Show that A, B and C^c are independent events.

(Scratch paper)