Please provide complete and well-written solutions to the following exercises.

Due May 17th, in the discussion section.

Homework 6

Exercise 1. Recall the Game of Chicken is defined as follows. Each player chooses to chicken out (C) by swerving away, or she can continue drive straight (D). Each player would prefer to continue driving while the other chickens out. However, if both players choose to continue driving, catastrophe occurs. The payoffs follow:

Player I \ Player II	C	D
C	$(6,6)$	$(2,7)$
D	$(7,2)$	$(0,0)$

Find all Nash equilibria for the Game of Chicken. Prove that these are the only Nash equilibria. Then, verify that

$$z = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & 0 \end{pmatrix}$$

is a Correlated Equilibrium. Can you find a Correlated Equilibrium such that both players have a payoff larger than 5? (Hint: when trying to find such a matrix z, assume that $z_{22} = 0$ and $z_{12} = z_{21}$.)

Exercise 2. In the Game of Chicken, you should have found only three Nash equilibria. Recall that any convex combination of Nash equilibria is a correlated equilibrium. However, the converse is false in general! We can see this already in the Game of Chicken. Show that the Correlated Equilibrium

$$z = \begin{pmatrix} 1/3 & 1/3 \\ 1/3 & 0 \end{pmatrix}$$

is not a convex combination of the Nash equilibria. Put another way, the payoffs from this Correlated Equilibrium cannot be found by randomly choosing among the Nash equilibria.

Exercise 3. Give an example of a two-person zero-sum game where there are no pure Nash equilibria. Can you give an example where all entries of the payoff matrix are different?

Exercise 4. Recall that the game of Rock-Paper-Scissors is defined by the payoff matrices

$$A = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix}, \qquad B = A^T.$$

Then the game is symmetric. (And also, note that $A+B=0$, so that the game is a zero-sum game.)

Show that $(1/3, 1/3, 1/3)$ is the unique Nash equilibrium. Then, show that this Nash equilibrium is **not** evolutionarily stable.

This observation leads to interesting behaviors in population dynamics. A certain type of lizard has three kinds of sub-species whose interactions resemble the Rock-Paper-Scissors game. The dynamics of the population cycle between large, dominant sub-populations of each of the three sub-species. That is, first the "Rock" lizards are a majority of the population, then the "Paper" lizards become the majority, then the "Scissors" lizards become the majority, and then the "Rock" lizards become the majority, and so on.