Department of Teaching and Learning, Steinhardt School of Culture, Education, & Human Development
Department of Mathematics, Courant Institute of Mathematical Sciences
New York University

MTHED-UE-1049: Mathematical Proof and Proving (MPP)
MATH-UA-125: Introduction to Mathematical Proofs

Instructors:
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Prof. Orit Zaslavsky, Dept. of Teaching & Learning

Spring, 2012
Monday 12:00-2:00pm, Kimmel Center, room 804

Course Description

The course introduces elements of mathematical proof, focusing on three main themes: 1. The meaning of mathematical statements – universal/existential; 2. The roles of examples in determining the validity of mathematical statements; 3. The various forms and methods of mathematical proofs, including direct (deductive) proof; proof by exhaustion; indirect proof (by contradiction, or by contrapositive); mathematical induction; disproof by counterexample. This is a problem-based course. Lessons are structured around activities that engage students in doing proofs that are meaningful to them and based on mathematical topics with which they are familiar.

Course Objectives

Students should be able to:

- Apply various methods of proving to mathematical statements that are taken from elementary number theory, geometry, basic combinatorics;
- Present proofs in an accurate and coherent way;
- Evaluate the validity of proofs (some with flaws).

Recommended Books (will be put on reserve in the library)

Chartrand, G., Polimeni, A. D., & Zhang, P. (2008). *Mathematical proofs: A transition to advanced mathematics.* NY: Pearson, Addison Wesley.

Fendel, D., & Resek, D. (1990). *Foundations of higher mathematics: Exploration and proof*. NY: Addison-Wesley Publishing Company.

Additional Resources

Jacobs, H. R. (1974). *Geometry.* NY: W. H. Freeman and Company. Chapter 1: *The nature of deductive reasoning* (pp. 8-70).

Movshovitz-Hadar, N., & Webb, J. (1998). *One Equals Zero and other mathematical surprises, paradoxes, fallacies, and mind bogglers*. Key Curriculum Press.

Nelson, R. B. (1993). *Proofs without words*. MAA.

Assignments

1. Weekly assignments:
   On a weekly basis, students will receive an assignment that (usually) has two parts:
   (i) A task requiring application of a proof method or principle that was illustrated and discussed in the previous class;
   (ii) A mathematical activity that prepares them for dealing with a method that will be introduced in the following class.
   Students will need to exhibit accurate and clear presentation of their claims.
2. Mid-term exam
3. Final exam

Assessment and Grading

Active participation in class activities and contribution to discussions and debates – 20%;
Weekly assignments – 40%;
Mid-term exam – 20%.
Final exam – 20%.

The grading of assignments and exams will be based on mathematical correctness and coherence of presentations.

In order to get credit for the course, you must attend at least 10 (of the 12) classes, and submit on time at least 80% of the weekly assignments. Late submissions may not be accepted. You also need to take both mid-term and final exams.

Grading Rubrics for participation (out of 20 points):

A **Excellent (20 points)**	**B** **Good (17 points)**	**C** **Adequate (14 points)**	**D** **Poor (10 points)**
Participates in class activities and contributes to discussions in every session.	Participates in class activities and contributes to class discussions in most sessions.	Participates in class activities and contributes to class discussions in some sessions.	Participates in class activities but makes no contributions or very few contributions to class discussions.

Note that points for unexcused absences may be deducted from the points in the above rubrics.

The final grade will be determined by the sum of points accumulated for each component (participation, weekly assignments, and final project), according to Steinhardt School of Education Grading Scale:

A	93-100
A-	90-92
B+	87-89
B	83-86
B-	80-82
C+	77-79

C	73-76
C-	70-72
D+	65-69
D	60-64
F	Below 60

Course Outline (tentative)

Week	Topic
Week 1	**Introduction to the course** **Pre-test**
Week 2	**Symbolic Logic** Converting statements (such as "For every, …", "There exists,…") into mathematical notations, thus making it clear what one has to prove; The negation of mathematical statements; The difference between logical implication and equivalence.
Week 3	**The roles of examples in proof and proving** The roles of examples in determining the validity of mathematical statements, such as: one supporting example is not sufficient for proving a valid universal statement while one supporting example is sufficient for proving a valid existential statement; Additional supporting examples are superfluous; One counter-example is sufficient for disproving a universal mathematical statement, but not an existential statement.
Week 4	**Direct Proof** (part 1) This is the most commonly used approach to proofs in mathematics. The students will learn to prove statements such as "A implies B", or equivalently, "not B implies not A", using inference rules (e.g., modus ponens and modus tollens).
Week 5	**Direct Proof** (part 2) Direct proof applied to elementary number theory.
Week 6	**Direct Proof** (part 3) Direct proof applied to geometry.
Week 7	**Mid-term Exam (3/19/2012)** **Proof by cases** Proof by exhaustion of all cases can be illustrated for several mathematical theorems, e.g., an inscribed angle in a circle is half the measure of its corresponding central circle; two integers are of the same parity if and only if their sum is even.
Week 8	**Mathematical Induction** (part 1) Introduction to the principle of mathematical induction, and its equivalence to the principle of the well ordered set of the Integers. Concepts, definitions, notations, and conventions relates to mathematical induction.
Week 9	**Mathematical Induction** (part 2) Mathematical induction applied to proving elementary properties of sums (finite and infinite).
Week 10	**Mathematical Induction** (part 3) Mathematical induction applied to proving inequalities (e.g., Jensen's inequality for discrete sets).

Week	Topic
Week 11	**Indirect Proof** (part 1) Proof by contradiction (i.e., assuming that a certain statement is false and from this assumption arriving at a statement that contradicts some assumption made in the proof or some know fact) is in some cases the only way to prove a statement (such as " $\sqrt{2}$ is an irrational number", or Cantor's diagonalization procedure for proving that the numbers in the interval [0,1] are not countable).
Week 12	**Indirect Proof** (part 2) Proof by contrapositive (i.e., building on the logical equivalence between "A implies B" and "Not B implies Not A") is often a useful method. To prove "If A then B" by the method of contrapositive means to prove "If Not Q, Then Not P".
Week 13	**Indirect Proof** (part 3) A direct approach to indirect proofs; e.g., the case of proving that there are an infinite number of primes (Leron, 1985).
Week 14	**Disproving** Disproving universal statements by generating counter-examples; Disproving existence statements by indirect methods. **Summing up and reflections on the course**

<u>Note</u> that throughout the course there will be opportunities to **Evaluate Proofs.** This part will help students develop an understanding of the consequences of the chain of mathematical statements that they make, i.e., what did they prove. We will discuss "faulty proofs" and learn to detect flaws in what may seem as a convincing proof.

Accommodation for NYU Students with Disabilities
Any student attending NYU who needs an accommodation due to a chronic, psychological, visual, mobility, and/or learning disability, or is Deaf or Hard of Hearing should register with the Moses Center for Students with Disabilities at 212 998-5980, 240 Greene Street. See www.nyu.edu/csd.

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You violate the principle of academic integrity when you:
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- Submit the same work for two or more different courses without prior permission from your professors;
- Receive help on a take-home examination that calls for independent work;
- Plagiarize.

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- Purchase documents;
- Report from other's oral work;
- Paraphrase or restate someone else's facts, analysis and/or conclusions;
- Copy directly from a classmate or allow a classmate to copy from you.

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