Department of Teaching and Learning, Steinhardt School of Culture, Education, & Human Development
Department of Mathematics, Courant Institute of Mathematical Sciences

MTHED-UE-1049: Mathematical Proof and Proving (MPP)
MATH-UA-125: Introduction to Mathematical Proofs

Homework No. 5

This homework should be submitted just before the beginning of class, on March 19th, 2012.
Please write in a black ink pen, so it is clear and easy to read! Use white paper!!
Write your name in Capital letters on the top of each page and number the pages.

1. In class we proved the 'triangle Inequality', that is, for any real numbers a,b the following inequality holds: $|a+b| \le |a|+|b|$. You may use this inequality to prove the following:

   Prove that for any real numbers a,b the following inequality holds: $|a|-|b| \le |a+b|$.

2. Which mean is larger? The ***Arithmetic Mean*** of a and b: $\frac{a+b}{2}$, or the ***Geometric Mean*** of a and b: $\sqrt{a \cdot b}$? Prove your claim real numbers a,b that satisfy $0<a<b$.

3. Definitions:

   $\frac{1}{\frac{1}{2}\cdot\left(\frac{1}{a}+\frac{1}{b}\right)}$ is called the ***Harmonic Mean*** of a and b (you can also write it as $\frac{2\cdot ab}{a+b}$. Why?)

   $\sqrt{\frac{a^2+b^2}{2}}$ is called the ***Root Mean Square*** of a and b.

   $\frac{b^2+a^2}{b+a}$ is called the ***Contraharmonic Mean*** of a and b.

   Which is the largest mean (of the five)? Which is the smallest? Can you order them from smallest to largest? How? Explore and formulate conjectures; try to prove or disprove some of your conjectures for real numbers a,b that satisfy $0<a<b$.

4. If $\frac{a}{b}$ and $\frac{c}{d}$ are positive fractions such that $0<\frac{a}{b}<\frac{c}{d}$, is $\frac{a+c}{b+d}$ an intermediate value?

   In other words, is $\frac{a}{b}<\frac{a+c}{b+d}<\frac{c}{d}$?

   Prove your claim.