Department of Teaching and Learning, Steinhardt School of Culture, Education, & Human Development
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MTHED-UE-1049: Mathematical Proof and Proving (MPP)
MATH-UA-125: Introduction to Mathematical Proofs

Homework No. 4

This homework should be submitted just before the beginning of class, on March 5th, 2012.
Please write in a black ink pen, so it is clear and easy to read! Use white paper!!
Write your name in Capital letters on the top of each page and number the pages.

1. In class we filled the following Truth Tables:

P	Q	$P\Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\sim P$	$\sim Q$	$(\sim Q)\Rightarrow(\sim P)$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

We saw that the implication $P\Rightarrow Q$ and its contrapositive $(\sim Q)\Rightarrow(\sim P)$ have the same Truth Table. This is another way of showing that they are equivalent. In terms of proving.

(i) Fill the following Truth Table, and based on it, find an equivalent statement to $P\Rightarrow Q$ and $(\sim Q)\Rightarrow(\sim P)$. Note that $P\vee Q$ is true whever P or Q or both P and Q are true.

P	Q	$P\vee Q$	$\sim P\vee Q$	$P\vee\sim Q$
T	T			
T	F			
F	T			
F	F			

(ii) Fill the following Truth Table, and based on it, find the negation of $P\vee\sim Q$. Note that $P\wedge Q$ is true only whever both P and Q are true.

P	Q	$P\wedge Q$	$\sim P\wedge Q$	$P\wedge\sim Q$
T	T			
T	F			
F	T			
F	F			

(iii) Fill the following Truth Table, and make some connections to the above Truth Tables (look for equivalent statements, or negations of statements).

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$P \Leftrightarrow Q$
T	T			
T	F			
F	T			
F	F			

2. In class we investigated properties of 3-digit numbers that are divisible by 37, and presented ideas for at least two ways to prove a conjecture you proposed.

   (i) Based on what we did in class, or any other ideas you have, write a full and well constructed proof to the following claim:

   If a 3-digit number is divisible by 37, then any 3-digit number that is obtained by a cyclic change of order of digits is also divisible by 37.
   (We can agree that when one of the digits is "0", we write it even when it is in the 10^2s place, e.g., 074).

   Note: When you write the proof, you should explicitly state the GIVEN and the RTP (what is Required To Prove). In your proof, you should indicate exactly where you used the given.

   (ii) Does your proof work for 4-digit numbers? Explain your answer.

3. For a and b that are positive real numbers, such that: $b>a>0$, prove that:

   (i) $a < \frac{a+b}{2} < b$

   (ii) $a < \sqrt{a \cdot b} < b$

   (iii) $\frac{a+b}{2}$ is the *arithmetic mean* of a and b, and $\sqrt{a \cdot b}$ is the *geometric mean* of a and b.
   Write in your own word what you actually proved in (i) and (ii) about these means.

4. Give the definition and examples for each of the following:

   (i) A prime number.

   (ii) A rational number.

   (iii) An irrational number.

5. (i) Is 1 a prime number? Explain.

   (ii) Is a product of two prime numbers a prime number? Explain.