

MTHED-UE-1049: Mathematical Proof and Proving (MPP)  
MATH-UA-125: Introduction to Mathematical Proofs

**Homework No. 2**

**This homework should be submitted just before the beginning of class, on February 13<sup>th</sup>, 2012.**

**Please write in a black ink pen, so it is clear and easy to read!**

**Write your name in Capital letters on the top of each page and number the pages.**

1. Let  $A = \{x \in \mathbb{R}, x > 3 \text{ or } x < 2\}$ ,  $B = \{x \in \mathbb{R}, x \leq 3 \text{ and } x > -1\}$ ,  $C = \{x \in \mathbb{R}, x^2 > 4\}$ ,  
 $D = \{x \in \mathbb{R}, x < 2 \text{ and } x^2 > 9\}$ ,  $\mathbb{R} = X$  (the universal set).

- (a) What are:  $A^c$ ?  $A \cup A^c$ ?  $B^c$ ?  $B \cup B^c$   
(b) What are:  $A \cup B$ ?  $A \cap B$ ?  
(c) Describe  $C$  in a way that is easy to represent on a number line.  
(d) Sketch a representation of  $C$  on a number line.  
(e) What is  $C^c$ ?  
(f) Describe  $D$  in a way that is easy to represent on a number line.  
(g) Sketch a representation of  $D$  on a number line.  
(h) What are:  $A \cap D$ ?  $A \cup D$ ?

2. For what values of  $x$ ,  $x \in \mathbb{R}$ , is the following (open) statement true?

$$\frac{x^2 - 9}{x + 3} = x - 3$$

Explain your answer.

3. Is the following statement true?

“For every  $y$ ,  $y \in \mathbb{R}$ , there exists an  $x$ ,  $x \in \mathbb{R}$ , such that:  $x^2 = y$ ”

Explain your answer.

4. For what values of  $x$  and  $y$ ,  $x, y \in \mathbb{R}$ , is the following (open) statement true?

$$x^2 = y$$

Explain your answer.

5. Examine the following statement: "If  $n$  is an even number, then  $(n^2 - 3n + 1)$  is positive".
- Give an example that contradicts the statement;
  - Give an example that does not contradict the statement;
  - Is the given statement true? Explain your answer.
  - What is the negation of this statement?
  - Is the negation true? Explain your answer.
6. Examine the following statement ( $x, y \in R$ ): "If  $x < 2$ , then  $y > -3$ ".
- What would be a counterexample to this statement?
  - What is the contrapositive of this statement?
  - What is the negation of this statement?
  - Is the negation true? Explain your answer.
7. Examine the following statement ( $x \in R$ ): "If  $x^2 + 1 < 0$ , then  $-x^2 - 3 < 0$ ".
- Is the premise true?
  - Is the conclusion true?
  - Is the statement true? Explain your answer.
8. Examine the following statement ( $x \in R$ ): "If  $x^2 + 1 < 0$ , then  $-x^2 - 3 > 0$ ".
- Is the premise true?
  - Is the conclusion true?
  - Is the statement true? Explain your answer.