Department of Teaching and Learning, Steinhardt School of Culture, Education, & Human Development
Department of Mathematics, Courant Institute of Mathematical Sciences

MTHED-UE-1049: Mathematical Proof and Proving (MPP)
MATH-UA-125: Introduction to Mathematical Proofs

Homework

This homework should be submitted just before the beginning of class, on Feruary 6th , 2012.
Please write in a black ink pen, so it is clear and easy to read!
Write your name in Capital letters on the top of each page and number the pages.

1. Let $A = \{1, 2, 5\}$ and $B = \{2, 7\}$ what are: $A \cup B$? $A \cap B$? $A \times B$?

2. Let $A = \{x \in N, \; x \text{ is a multiple of } 6\}$, $B = \{x \in N, \; x \text{ is a multiple of } 15\}$, and $N = X$ (the universal set). What are: A^c? $\left(A^c\right)^c$? B^c? $A \cup B$? $A \cap B$? $A^c \cup B^c$? $A^c \cap B^c$? $\left(A \cap B\right)^c$? $\left(A \cup B\right)^c$? $A^c \cup B$? $A \cup B^c$?

3. Let $A = \{x \in N, \; x \text{ is a multiple of } 6\}$, $B = \{x \in N, \; x \text{ is a multiple of } 3\}$, and $N = X$ (the universal set). What are: A^c? $\left(A^c\right)^c$? B^c? $A \cup B$? $A \cap B$? $A^c \cup B^c$? $A^c \cap B^c$? $\left(A \cap B\right)^c$? $\left(A \cup B\right)^c$? $A^c \cup B$? $A \cup B^c$? $\left(A^c \cup B\right)^c$? $\left(A \cup B^c\right)^c$? $A^c \cap B$? $A \cap B^c$?

4. Let $A = \{x \in N, \; x \text{ is a multiple of } 5\}$, $B = \{x \in N, \; x \text{ is a multiple of } 3\}$, and $N = X$ (the universal set). What are: A^c? $\left(A^c\right)^c$? B^c? $A \cup B$? $A \cap B$? $A^c \cup B^c$? $A^c \cap B^c$? $\left(A \cap B\right)^c$? $\left(A \cup B\right)^c$? $A^c \cup B$? $A \cup B^c$?

5. Try to prove that: $\left(A \cap B\right)^c = A^c \cup B^c$. Share your thinking on how to approach this problem.

6. Prove that: If p and q are two odd numbers, then $(p+q)\cdot(p-q)$ is a multiple of 4.
   Write a full proof in an acceptable form.

7. Assume that we know <u>for sure</u> that: **All Pelicans Eat Fish** (i.e., this is a true statement).

   (a) Which of the following statements follow from the above statement?

   (b) Which of the following statements cannot be true, if the above statement is true?

      (*i*)   If a bird is a Pelican, then it eats fish.
      (*ii*)   If a creature eats fish, then it is a Pelican.
      (*iii*)   If a bird is not a Pelican, then it does not eat fish.
      (*iv*)   If a creature does not eat fish, then it is not a Pelican.
      (*v*)   If a bird is a Pelican, then it does not eat fish.
      (*vi*)   If a creature does not eat fish, then it is a Pelican.

8. Assume that we know <u>for sure</u> that: **In a far away country named *Shesing* all woman who live there can sing** (i.e., this is a true statement).

    (a) Which of the following statements follow from the above statement?

    (b) Which of the following statements cannot be true, if the above statement is true?

    (*i*)   If you live in *Shesing* and are a woman, then you can sing.

    (*ii*) If you live in *Shesing* and can sing, then you are a woman.

    (*iii*) If you live in *Shesing* and are not a woman, then you cannot sing.

    (*iv*) If you live in *Shesing* and cannot sing, then you are not a woman.

    (*v*)   If you live in Shesing and are a woman, then you cannot sing.

    (*vi*) If you live in *Shesing* and cannot sing, then you are a woman.

9. For each of the following statements determine whether is always true, sometimes true, or never true:

    (*i*)   When you add two rational numbers, you get the same result as when you multiply them.

    (*ii*) $(a+b)^2 = a^2 + b^2$, when $a, b \in R$.