Please provide complete and well-written solutions to the following exercises.

(No due date, though the quiz on February 10th or 12th will be based on this homework.)

Assignment 6

Exercise 1. Determine whether or not the following field is conservative. $F(x,y,z) = (y,(x+z),-y)$.

Exercise 2. Let $F(x,y,z) = (y,x,z^3)$ be a vector field. Either show that F is not conservative, or find a function $f\colon \mathbf{R}^3 \to \mathbf{R}$ such that $\nabla f = F$.

Exercise 3. Let $F(x,y,z) = (y\sin z, x\sin z, xy\cos z)$ be a vector field. Find a function $f\colon \mathbf{R}^3 \to \mathbf{R}$ such that $\nabla f = F$.

Exercise 4. Let $F(x,y,z) = (yz,xz,xy)$ be a vector field. Show that F is conservative. Then, evaluate the line integral of F from the endpoint $(1,1,2)$ to the endpoint $(3,5,2)$.

Exercise 5. Let $F(x,y,z) = (x^2+y, y^2+x, ze^z)$ be a vector field. Find the work done by F along the following paths from $(1,0,0)$ to $(1,0,1)$.

- The straight line segment where $x=1$, $y=0$ and $0 \leq z \leq 1$.
- The helix $s(t) = (\cos t, \sin t, t/(2\pi))$ where $0 \leq t \leq 2\pi$.
- The straight line from $(1,0,0)$ to $(0,0,0)$, followed by the parabola $z=x^2$, $y=0$ from $(0,0,0)$ to $(1,0,1)$.

Exercise 6. Let $F(x,y) = \nabla(x^3y^2)$ be a vector field in the plane. Let C be the path in the xy plane from $(-1,1)$ to $(1,1)$ that consists of the line segment from $(-1,1)$ to $(0,0)$, followed by the line segment from $(0,0)$ to $(1,1)$. Evaluate $\int_C F\cdot T\,ds$ in the following two ways.

- Find parametrizations for the segments involved in the definition of C, and evaluate the resulting line integrals directly.
- Use the fact that $f(x,y) = x^3y^2$ satisfies $\nabla f = F$.

Exercise 7. Let $F(x,y)$ be a vector field in the plane, defined as follows

$$F(x,y) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right).$$

Note that F is undefined at $(0,0)$.

- Verify that F satisfies the cross partial test for conservative vector fields. Can we conclude that F is conservative?
- Find a function f on the plane such that $\nabla f = F$, thereby showing that F is conservative.

- Do these results contradict the following theorem from the book? (Theorem 4) Let F be a vector field on a simply connected domain D. If F satisfies the cross-partials condition then F is conservative. (Explain your reasoning.)

Exercise 8. Consider the vector field $F(x, y, z) = (x, -z, y)$ on $\mathbf{R}^3$. Show that F is not conservative in the following three ways.

- Use the cross partial test.
- Find a closed curve C such that F has a nonzero line integral on C.
- Find two paths between the same pair of points such that the line integral is different on each path.