Please provide complete and well-written solutions to the following exercises.

(No due date, though the quiz on January 13th or 15th will be based on this homework.)

Assignment 2

Exercise 1. Using polar coordinates (r, θ), plot the function $r^2 = \sin\theta$ for $\theta \in (0, \pi)$. The end result should resemble a figure eight.

Exercise 2. Using polar coordinates (r, θ), plot the function $r = 1 + \cos\theta$. The end result should resemble an apple, or a heart.

Exercise 3. Using polar coordinates (r, θ), plot the function $r = (1/2) + \cos\theta$.

Exercise 4. Using polar coordinates (r, θ), plot the functions $r = \sqrt{2}$ and $r^2 = 4\sin\theta$, and label their points of intersection.

Exercise 5. Change from Cartesian coordinates to polar coordinates in order to evaluate the following integral.

$$\int_{y=0}^{y=5} \int_{x=0}^{x=y} x dx dy.$$

Exercise 6. Change from Cartesian coordinates to polar coordinates in order to evaluate the following integral.

$$\int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{1-(x-1)^2}} \frac{y}{x^2+y^2} dy dx.$$

Exercise 7. Find the value of a which maximizes the volume of the region in the first octant bounded by $x = a$, $y = 1 - a$, and $z = xy^2$. (As usual, the first octant refers to the region in Euclidean space where $x \geq 0$, $y \geq 0$ and $z \geq 0$.)

Exercise 8. Evaluate the following integral

$$\int_{z=-1}^{z=1} \int_{x=-1}^{x=1} \int_{y=-1}^{y=1} (x + y + z) dy dx dz.$$

Exercise 9. Sketch the region of integration of the following integral, and then evaluate the integral.

$$\int_{x=0}^{x=1} \int_{y=-1}^{y=0} \int_{z=0}^{z=y^2} dz dy dx.$$

Exercise 10. Using a triple integral, find the volume of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 3)$.

Exercise 11. Find the volume of the region that is bounded by the cylinder $x^2 + y^2 = 4$, by the plane $z = 0$, and by the plane $x + z = 3$.

Exercise 12. Evaluate the following integral, by changing the order of integration.

$$\int_{z=0}^{z=1}\int_{x=0}^{x=1}\int_{y=x^2}^{y=1} 24xze^{zy^2}dydxdz.$$