Digest 5, Homework 5

(A compilation of emailed homework questions, answered around Wednesday.)

Question. [Exercise 2]

Find the line integral of the function $f(x, y, z) = x + y + z$ over the straight line segment from $(1, 2, 3)$ to $(0, -1, 1)$.

(From a Student): How should this problem be approached if we are not given a parametrization $c(t)$ for the line segment? Without the parametrization, I don't see a way to get the boundaries for the integral.

Answer. This is part of the problem. You need to find the parametrization of the straight line segment from $(1, 2, 3)$ to $(0, -1, 1)$. This should have been a topic covered in 32A (Curves defined by parametric equations). As a warmup, try solving the simpler problem: what is a parametrization of a line starting at the point $(1, 0, 0)$ and ending at $(0, 0, 0)$?

Question. (From a Student): For question 7, I got a negative answer. Can line integrals of vector fields be negative? (Negative work?)

For question 8, I got similar but different answers for the two parts (changing directions). Is that supposed to happen? I expected them to be the same answer but with one negative.

Answer. In question 8, you are asked to compute $\int_C F \cdot T ds$, and then to compute the same quantity with the parametrization reversed. As discussed in class, reversing the direction will change the sign of $\int_C F \cdot T ds$, basically because reversing the direction multiplies the tangent vector T by -1. So, each answer should be the negative of the other. This also answers your first question. If I change the direction of any line integral, then I change its sign, so the line integral of a vector field can be positive or negative (or zero).

Question. For question 10, I got a $n(t) = (3, 3)$, but the question says moving "upward" over C. Does $(3, 3)$ count as "upward" since it's positive? (It's not strictly north though...)

When do we need to worry about picking the right direction of the two possible choices?

Do you recommend drawing a picture for line integral questions to see if my answers are reasonable?

Answer. I agree that $(3, 3)$ is not pointing strictly north. However, there are only two choices of the normal vector in this case. One choice has positive y-component, and the other has negative y-component. Moving upward suggests having positive y-component, so it seems you have made the correct choice. In general, the direction of the normal vector will always be specified, so you always need to worry about choosing the correct direction to adhere to the specifications of the problem.

You could draw pictures, and it could help for simpler vector fields, but for a complicated vector field (or a complicated curve), such a drawing would be difficult to do, so it may not be so enlightening.

Question. (From a student): Can you please explain the physical meaning of line integrals over a function/vector field and flux integrals, and perhaps an example of each?

The only one I understand is a line integral over a function of two variables, which I found on wikipedia. I'm confused on what a line integral over a function of three variables represents.

Answer. If the vector field F is a force field (e.g. if it shows the direction and magnitude of the force of gravity of Earth), then $\int_C F \cdot T\, ds$ represents the work done against gravity as we travel along the path C. In this example, we are integrating the function $F \cdot T$ over the path C (though, as we discussed, we can integrate any function on a curve). If the vector field F is the flow field for some fluid in two dimensions (e.g. if it shows the direction and magnitude of the flow of water or air in a two-dimensional channel), then $\int_C F \cdot e_n\, ds$ represents the total flow of fluid that is passing through the curve C, in the direction e_n. In this case, we are integrating the function $F \cdot e_n$ over the curve C.

Question. (From a Student): Every time we write a flux integral, how to decide the direction of $n(t)$? In your note, you assume s traverses C in the counterclockwise direction, but what if we traverse C in the clockwise direction, and what if C is a straight line, as is the case in Exercise 9 and 10? And do we need to know how to compute it in $\mathbf{R}^3$?

Answer. In an exercise (as in the case on this homework), I will typically specify the direction of the normal vector. For example, in Exercise 9, we choose the normal vector that is pointing in the exterior direction of the triangle. And in Exercise 10, we choose the normal vector that is pointing upwards (i.e. the normal vector with positive y component.) As long as we choose the normal vector to point in the requested direction, it doesn't matter whether C is traversed in the clockwise or counterclockwise direction. In fact, we can even compute the flux over a curve that is not necessarily closed, as we discussed in class, and in Example 5.18 in the notes.

As for computing the flux in $\mathbf{R}^3$, that is a good question. If you think about it, there is no suitable definition of the flux out of a curve in 3-dimensions. (For example, to return your own question at you: which normal do you choose? For a curve in three-dimensions, there are infinitely many normal vectors to choose from!) However, we will soon be computing the flux in $\mathbf{R}^3$ through surfaces, and here we can make a suitable definition of the flux through a surface, since there are only two normal directions on a surface.

Question. How can I avoid making "careless" mistakes on the exams?

Answer. I'm not sure about that one since I struggle with that too; I guess just check your work as best you can, give yourself time to check the work as much as possible, and try to develop a good sense of self-doubt (hopefully you practice this on the homeworks). That is, try to always ask yourself "What if I am totally wrong?" or "Can I solve this problem in a different way that gets the same answer?" or "Can I do some quick verifications that convince me that this answer is correct," etc.