Please provide complete and well-written solutions to the following exercises.

Assignment 8

Due November 20, at the beginning of class.

Exercise 1. Consider the unit sphere $x^2 + y^2 + z^2 = 1$. Let (a, b, c) be a point in the unit sphere. Find an equation for the tangent plane to the sphere at an arbitrary point (a, b, c), using the tangent plane for implicitly defined surfaces.

Exercise 2. Consider the following surface: the set of $(x, y, z) \in \mathbf{R}^3$ such that

$$(x^2 + y^2 + z^2 + 3)^2 = 16(x^2 + y^2).$$

This surface is known as a torus, or the surface of a donut.

Find an equation for the tangent plane to the torus at the point $(1, 0, 0)$.

Exercise 3. Let $f(x, y, z) = x^2 + y^2 + z^2$ and let $F(t) = \sin(t)$. Compute $\nabla F(f(x, y, z))$.

Let $g(x, y, z) = e^{x^2} y + z$. Compute $\nabla(f \cdot g)(x, y, z)$.

Let $v = (1, 1, 2)$. Compute $D_v g(0, 1, 3)$.

Let $v = (1, 1, 1)$. Compute $D_v f(0, 1, 0)$, $D_v f(1, 0, 0)$ and $D_v f(1, 1, 1)$.

Exercise 4. It is the zombie apocalypse. It is safer at the moment to run to higher ground. The height of the land nearby is proportional to the function $f(x, y) = e^{-(x^2+y^2)/2} + xy^3$. You are located at the point $(x, y) = (1, -1)$. In which direction should you run if you want to immediately:

- Move to higher ground (increasing your height as quickly as possible)?
- Stay at the same elevation?
- Move to lower ground (decreasing your height as quickly as possible)?

Exercise 5. Suppose we know that a level surface of a function $f(x, y, z)$ is given by the surface $x^2 + y^2 + z^2 = 1$. Let $v = \nabla f(1, 0, 0)$. Is it true that $v/\,||v|| = (1, 0, 0)$? Justify your answer.