
Please provide complete and well-written solutions to the following exercises.

Assignment 5

Due October 30, at the beginning of class.

Exercise 1.

- Sketch the domain in \mathbf{R}^2 which is the set of all (x, y) such that $1 \leq x^2 + y^2 \leq 4$.
- Sketch the domain in \mathbf{R}^3 which is the set of all (x, y, z) such that $0 \leq x \leq 1$, $0 \leq y \leq 1$ and $0 \leq z \leq 1$.
- Sketch the domain in \mathbf{R}^3 which is the set of all (x, y, z) such that $x^2 + y^2 \leq 4$, $z \geq 0$ and $y + z \leq 5$.

Exercise 2.

- Sketch the function $z = f(x, y) = x^2 - y^2$.
- Sketch the function $z = f(x, y) = e^{-(x^2+y^2)}$.
- Sketch the function $z = f(x, y) = 1/(xy)$.

Exercise 3. Compute the following limit, or show that the limit does not exist.

$$\lim_{(x,y) \rightarrow (0,1)} \frac{x}{y}.$$

Exercise 4. Assume that $\lim_{(x,y) \rightarrow (0,0)} f(x, y) = 3$ and $\lim_{(x,y) \rightarrow (0,0)} g(x, y) = 2$. Compute the following quantities:

- $\lim_{(x,y) \rightarrow (0,0)} 2f(x, y) + \lim_{(x,y) \rightarrow (0,0)} g(x, y)$.
- $[\lim_{(x,y) \rightarrow (0,0)} f(x, y)][\lim_{(x,y) \rightarrow (0,0)} g(x, y)]$.
- $\lim_{(x,y) \rightarrow (0,0)} \cos(f(x, y))$.

Exercise 5. Compute the following limit:

$$\lim_{(x,y) \rightarrow (0,3)} (1+x)^{y/x}.$$

Exercise 6. Consider the following function $f: \mathbf{R}^2 \rightarrow \mathbf{R}$.

$$f(x, y) = \begin{cases} |y/x^2| e^{-|y/x^2|}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}.$$

Is f continuous or discontinuous at $(0, 0)$? Justify your answer.