Please provide complete and well-written solutions to the following exercises.

Assignment 4

Due October 23, at the beginning of class.

Exercise 1. Find a parametrization for the circle of radius 3 with center $(1, 2, 4)$ which is parallel to the xz-plane.

Exercise 2. Find a parametrization for the intersection of the cylinder $x^2 + y^2 = 1$ with the cylinder $y^2 + z^2 = 1$. (Make sure to parametrize the entire curve, and specify the domain of your parameter.)

Exercise 3. Find a parametrization for the intersection of the cone $x^2 + y^2 = z^2$ with the parabolic cylinder $y = z^2$. (Make sure to parametrize the entire curve, and specify the domain of your parameter.)

Exercise 4. The cone $x^2 + y^2 = z^2$, $z \geq 0$ has its bottom chopped off by the plane $z = 1$, resulting in the surface $x^2+y^2 = z^2$, $z \geq 1$, which is a cone with a hole in it. An egg is dropped into the top of the cone. The egg has the same shape as the ellipsoid $\frac{x^2}{2} + 2y^2 + 3z^2 = 1$. Is it possible for the egg to fit through the hole in the cone?

Exercise 5. Let $r(t) = (t^2, t+3, \tan^{-1}(t))$. Find the tangent line to the parametrized curve $r(t)$ at $t = 3$.

Exercise 6. Suppose a particle has position $r(t)$ at time t, where $r'(t) = (\frac{t}{t^2+1}, t, 1)$. Assume that $r(0) = (0, 0, 1)$. Find $r(1)$.

Exercise 7. Suppose a particle has position $r(t) = (1, t, (2/3)t^{3/2})$ at time $t \geq 0$. Find the arc length parametrization of the parametrized curve.

Exercise 8. Suppose a baseball is thrown from a height of 10 meters above the ground, with an initial velocity (in meters per second) of $v_0 = (5, 3, 2)$. How long does the baseball take until it hits the ground? (Ignore air friction.)

Exercise 9.

- Using Kepler's Third Law, show that if a planet revolves around a star with a period T and semimajor axis a, then the star has mass $M = 4\pi^2 a^3/(GT^2)$. That is, the data T and a alone can be used to determine the mass of the star.
- Show that if a planet revolves around a star of mass M in a circular orbit of radius R with speed v, then $M = Rv^2/G$.
- The sun revolves around the center of the Milky Way galaxy in an approximately circular orbit of radius $R \approx 2.8 \times 10^{17}$ km and velocity $v \approx 250$ km/s. Estimate the mass of the portion of the Milky Way galaxy that sits inside the orbit of the sun. (You may treat this portion of the Milky Way galaxy as a point mass at the center of the galaxy.)