Please provide complete and well-written solutions to the following exercises.

Assignment 3

Due October 16, at the beginning of class.

Exercise 1. Find the distance from the point $w = (1, 2, 3)$ to the line parametrized by $s(t) = (t, 2t, 1 + t)$, $-\infty < t < \infty$.

Exercise 2. Plot the parametrized curve in the plane: $s(t) = (\cos^3 t, \sin^3 t)$, $0 \leq t \leq \pi$.

Exercise 3. Find the distance of the point $(1, 2, 5)$ from the plane $2x + y - z = 0$.

Exercise 4. Find the angle between the planes $x + 2y + 3z = 1$ and $2x - y - 3z = 0$. Then, find a parametrization for the line of intersection of these planes.

Exercise 5. Suppose we have two parallel planes $ax + by + cz = d_1$ and $ax + by + cz = d_2$. Show that the distance between these two planes is

$$\frac{|d_1 - d_2|}{||(a, b, c)||}.$$

Exercise 6. Recall that if we have a point $w \in \mathbf{R}^3$ and a plane $ax + by + cz = 0$, then the distance from w to the plane is $|w \cdot (a, b, c)| \,/\, ||a, b, c||$.

- Using this fact, show that the distance of a point $w \in \mathbf{R}^3$ to the plane $ax+by+cz = d$ is
$$\frac{|-d + w \cdot (a, b, c)|}{||(a, b, c)||}.$$
- Find an equation for the sphere that is tangent to the planes $x + y + z = 3$ and $x + y + z = 9$, given that the center of the sphere lies inside the planes $2x - y = 0$ and $3x - z = 0$.

Exercise 7. For each of the following equations, identify the type of quadric surface that appears. Then, sketch the surface.

- The set of all $(x, y, z) \in \mathbf{R}^3$ such that $z = x^2$.
- The set of all $(x, y, z) \in \mathbf{R}^3$ such that $x^2 + 4y^2 + 9z^2 = 1$.
- The set of all $(x, y, z) \in \mathbf{R}^3$ such that $x^2 + y^2 - z^2 = 1$.
- The set of all $(x, y, z) \in \mathbf{R}^3$ such that $x^2 + y^2 = 2z^2$, and such that $z \geq 0$.