
Please provide complete and well-written solutions to the following exercises.

No due date, but the quiz in Week 2 in the discussion section (on October 6th or 8th) will be based upon this homework.

Assignment 2

Exercise 1. Let u, v, w be vectors in \mathbf{R}^3 . Assume that $u \cdot v = u \cdot w$. Is it true that $v = w$? Either explain why this is true, or find a counterexample.

Exercise 2. Let u, v be orthogonal vectors. Show that u, v satisfy the Pythagorean Theorem: $\|u + v\|^2 = \|u\|^2 + \|v\|^2$. Then, find vectors r, s such that $\|r + s\|^2 \neq \|r\|^2 + \|s\|^2$.

Exercise 3. Let $u = (1, 2, 3)$ and let $v = (0, 1, 2)$. Write u as a sum of two vectors $u = r + s$, where r is parallel to v , and s is orthogonal to v .

Exercise 4. In this problem, we can represent forces as vectors. For example, if a 150 pound person is standing at rest on level ground, then the force the person exerts on the ground is a vector of length 150 pointing into the ground. And the force the ground exerts on the person is a vector of length 150 pointing out of the ground. When all forces are in equilibrium, the sum of all vector forces is zero.

A rope is hanging horizontally between two posts. A 150 pound person is hanging from the rope, causing the rope to bend. To the left of the person, the rope is pointing at an angle of $\pi/3$ from vertical. To the right of the person, the rope is pointing at an angle of $\pi/4$ from vertical. What is the force that the left side of the rope is exerting on the person, and what is the force that the right side of rope is exerting on the person?

Exercise 5. A car at rest on level ground has a force due to gravity of 20,000 Newtons. This force is represented as a vector pointing into the ground with length 20,000. We want to push the car up a frictionless, three degree incline. How much force is required the push the car up the incline? (By pushing the car, we are applying a force parallel to the incline.)

Exercise 6. Using the definition of the cross product, verify the following standard identities:

$$(1, 0, 0) \times (0, 1, 0) = (0, 0, 1), \quad (0, 1, 0) \times (0, 0, 1) = (1, 0, 0), \quad (0, 0, 1) \times (1, 0, 0) = (0, 1, 0).$$

$$(1, 0, 0) \times (1, 0, 0) = (0, 1, 0) \times (0, 1, 0) = (0, 0, 1) \times (0, 0, 1) = (0, 0, 0).$$

Exercise 7. A tetrahedron has vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$. This tetrahedron has four faces. Find the outward pointing unit normal vectors to each face of the tetrahedron.

Exercise 8. Compute the area of the parallelogram defined by the vectors $v = (1, 2)$ and $w = (2, 3)$. Sketch the parallelogram.

Compute the volume of the parallelepiped defined by the vectors $u = (1, 2, 0)$, $v = (2, 3, 0)$ and $w = (0, 1, 3)$. Sketch the parallelepiped.

Compute the volume of the parallelepiped defined by the vectors $u = (1, 2, 3)$, $v = (1, 2, 0)$ and $w = (0, 0, 3)$.

Exercise 9. Find the area of the triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$, and $(2, 3, 4)$. (Hint: a triangle is “half” of a parallelogram.)

Exercise 10. Is it possible to find a vector $v \in \mathbf{R}^3$ such that $v \times (1, 0, 0) = (1, 1, 1)$? Either find the vector v , or explain why it is impossible to find such a vector v .

Exercise 11. Let v, w be two velocity vectors. Let c denote the speed of light. Suppose I am running at the velocity v . From my frame of reference, I throw a ball with velocity w . In classical physics, the velocity of the ball with respect to the earth is $v + w$. However, this is not exactly true. Einstein’s special theory of relativity provides a correction to this addition law. The velocity of the ball with respect to the earth is actually

$$\frac{v + w}{1 + \frac{v \cdot w}{c^2}} + \frac{1}{c^2} \cdot \frac{\gamma}{\gamma + 1} \cdot \frac{v \times (v \times w)}{1 + \frac{v \cdot w}{c^2}}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v \cdot v}{c^2}}}.$$

Suppose $v = (4c/5, 0, 0)$ and $w = (4c/5, 0, 0)$. Compute the velocity vector of the ball with respect to the earth, and also compute its length. (It is okay to leave your answer in terms of c .)

Suppose $v = (4c/5, 0, 0)$ and $w = (0, 4c/5, 0)$. Compute the velocity vector of the ball with respect to the earth, and also compute its length. (It is okay to leave your answer in terms of c .)