Digest 7

(A compilation of emailed homework questions, answered around Wednesday.)

Question. (From a student): The book defines the mixed partial derivative as $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$. However, you instead define $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$. Which definition should I use?

Answer. From Clairaut's Theorem, it really does not matter which definition you use (typically). For a typical function, both definitions coincide. If you want to pick one definition, just use the one we use in the notes. (I should mention that some books use Rogawski's convention, and other books use my convention). But I will never ask you a question where your choice of definition matters. For example, the last question on the homework just asked to show that the mixed partials are not equal. Answering this question correctly does not depend on which definition of a mixed partial you use.

Question. (From a student): On the quiz, I interpreted the expression $u^2/v + vxyz + \sin(xwv)$ as $u^2/(v + vxyz + \sin(xwv))$. Will I be penalized for this mistake?

Answer. In this case, you will not be penalized. If you made this mistake, the grader will act as if the function was the second expression. However, if this happens again, this mistake will not be okay, and you will be penalized. Order of operations is a standard thing, so in the future it would be good to remember this, but just this once you are okay.

What do I mean by order of operations? Well, if you type $1/1 + 2$ into Matlab (or Google, etc.), you will get 3. That is, it is a standard thing to multiply or divide first, and then add or subtract. I will try to avoid this notation in the future, but if it happens again, please remember to use order of operations.

Question. (From a student): Is there a relation between the implicitly defined tangent plane, and the formulas for the tangent plane from before?

Answer. The implicit formula is almost identical to the formula for the linearization of a function $f\colon \mathbf{R}^3 \to \mathbf{R}$. However, the formula for the tangent plane for the surface $g(x, y, z) = d$ is a bit different than the formula for the tangent plane $z = L(x, y)$ for a function $f\colon \mathbf{R}^2 \to \mathbf{R}$. However, the two formulas can be related in the following scenario. Suppose we can write g in the form $g(x, y, z) = -z + f(x, y)$, and if we consider the surface $g(x, y, z) = -z + f(x, y) = 0$, then the tangent plane for the implicitly defined surface $g(x, y, z) = 0$ becomes

$$\begin{aligned} 0 &= ((x, y, z) - (a, b, c)) \cdot \nabla g(a, b, c) = ((x, y, z) - (a, b, c)) \cdot (f_x(a, b), f_y(a, b), -1) \\ &= ((x, y) - (a, b)) \cdot \nabla f(a, b) + c - z = ((x, y) - (a, b)) \cdot \nabla f(a, b) - z + f(a, b) \end{aligned}$$

In the last line, we used $g(a, b, c) = 0 = -c + f(a, b)$, so $c = f(a, b)$. In summary, the implicitly defined tangent plane for g reduces to the formula for the tangent plane for f.

However, we are treating a surface in two different ways, so don't get confused. On the one hand, the surface is treated as a level surface of a three variable function g. On the other hand, the surface is written as $z = f(x, y)$ where f is a two variable function. In cases where it is difficult or impossible to solve for z as a function of x and y, it may be difficult or impossible to write the surface as $z = f(x, y)$. So, often it is better to look at the surface as the level surface of a three variable function.

Question. (From a student): For the second mid term, will we be tested on the Curvature?

Answer. This topic occurred in the range of topics covered by the midterm, so it is fair game, i.e. a curvature question could appear on the midterm.