Digest 4

(A compilation of emailed homework questions, answered around Wednesday.) (Sending this out Sunday, should update in one or two days.)

Question. (From a student): Regarding the midterm, should we be familiar with i, j, and k notation? I remember you mentioning in class that you don't like this notation but on the practice midterm there are a few questions that use it. Also, should we know how to find slopes of curves at certain points and how to parameterize intersections of curves?

Answer. As stated in class, I will never use i,j,k notation myself, and this includes all exams and homeworks. As for the other two topics, I don't think we covered those on or before October 9th, so they will not be on the exam. (Of course, we did cover the parametrization of intersections of surfaces.)

As mentioned in class, and in the email announcement, the exam only covers things we have done on or before the section on quadric surfaces.

Question. [Exercise 4] The cone $x^2 + y^2 = z^2$, $z \geq 0$ has its bottom chopped off by the plane $z = 1$, resulting in the surface $x^2 + y^2 = z^2$, $z \geq 1$, which is a cone with a hole in it. An egg is dropped into the top of the cone. The egg has the same shape as the ellipsoid $\frac{x^2}{2} + 2y^2 + 3z^2 = 1$. Is it possible for the egg to fit through the hole in the cone?

(From a student): Must the egg be dropped into the cone by exactly its position decided by the equation shown in space coordinate system, or is it okay to roll it to fit the hole?

Answer. The egg can be translated or rotated in any way to try to fit it through the opening in the cone.

Question. [Exam 1, Question 3b] (From a student): It said on the test the equation is $z^2 - 2y^2 + 10z^2 = 0$. It should be a cylinder since it does not depend on x. However, on the answer sheet, it says this should be a cone.

Answer. In Franz, I left the question as it was stated, and in Perloff they changed the question to $x^2 - 2y^2 + 10z^2 = 0$. The formula $x^2 - 2y^2 + 10z^2 = 0$ is a cone. However, the formula $z^2 - 2y^2 + 10z^2 = 0$ could be interpreted as either a (degenerate) cone, a cylinder, or a (degenerate) hyperboloid. So, there are actually three possible right answers in this case.

Question. [Exam 1, Question 5] (From a student): Isn't the cross product associative in the following way: $u \times (v \times w) = (u \times v) \times w$? I thought I got the right answer for question 5 using this identity.

Answer. No, this is incorrect.

In general, $u \times (v \times w) \neq (u \times v) \times w$. That is, the cross product is not associative. We can see this by choosing certain vectors as follows:

$$(1,1,0) \times [(1,0,0) \times (0,1,0)] = (1,1,0) \times (0,0,1) = (1,-1,0)$$

$$[(1,1,0) \times (1,0,0)] \times (0,1,0) = (0,0,-1) \times (0,1,0) = (1,0,0).$$

Question. [Exam 1, Question 5] (From a student): I thought that $u \times v$ is parallel to $u \times w$? This way, $(u \times v) \times (u \times w) = (0,0,0)$, so that the end answer is the zero vector

Answer. No, this is incorrect.

In general, $u \times v$ is not parallel to $u \times w$. So, this way of doing the problem is incorrect. This can be seen e.g. by explicitly calculating the vectors in the problem

$$(1,2,1) \times (3,4,1) = (-2,2,-2)$$

$$(1,2,1) \times (5,7,9) = (11,-4,-3)$$

Question. [Exam 1, Question 5] (From a student): About the Question 5 on the exam, I got the right answer using the easy way, but when I checked my answer with the regular cross product process I got the wrong answer due to miscalculation. I wrote both two answers on the sheet, so how many points can I get for this?(I have written explicitly the process of regular cross product.)

Answer. In the case that an exam response provides two self-contained answers, and if there is no indication by the student of which answer was preferred, then each response will be graded separately, and the average of these two scores is given for the question.

Question. [Exam 1, Question 5] (From a student): For the last question, I solved the matrices explicitly and got the answer wrong, is it possible for me to get partial credit for the matrixes that I solved right?

Answer. The last question explicitly stated that if you compute the cross products explicitly and you get the wrong answer, then you would receive at most half credit on that question. For people who used this approach, partial credit was given according to the correctness of the calculations, and I would estimate that the median score on this problem for people who took this approach and got the wrong answer is around 2 or 3 out of 10.

Question. [Exam 1, Question 4] (From a student): For the cross product, is it only existing in three dimensions or can it be generalized to higher dimensions? If the matrix calculation limits it to $\mathbf{R}^3$, does the property / geometrical meaning of "the magnitude of the cross product equals the magnitude of the two vectors crossing and the sine of the angle between them" only apply to $\mathbf{R}^3$ as well?

Answer. As far as this course is concerned, the cross product can only be defined for vectors in $\mathbf{R}^3$.

Here is an answer though that will talk about stuff outside of our course material:

It is possible to define a product on vectors in other dimensions, i.e. you could come up with some way of combining two vectors in $\mathbf{R}^4$ and outputting some other vector in $\mathbf{R}^4$, but whatever product definition you come up with cannot satisfy all of the properties of the usual cross product. The same goes for $\mathbf{R}^5$ and $\mathbf{R}^6$. Rather surprisingly, it is possible to define a kind of cross product for vectors in $\mathbf{R}^7$, which satisfies all of the "usual" properties of cross product. (The multiplication table is a bit formidable.) You could read about it here if you are curious. However, this is a strange exception, since you cannot define any cross product in $\mathbf{R}^8$ or $\mathbf{R}^9$, or $\mathbf{R}^{10}$, $\mathbf{R}^{11}$, etc.