Please provide complete and well-written solutions to the following exercises.

Due May 29, at the beginning of class.

Assignment 9

Exercise 1. Using the Ratio Test, determine whether the series converges, diverges, or that the test is inconclusive.

- $\displaystyle\sum_{n=1}^{\infty} n!e^{-n}$.
- $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$.

Exercise 2. For the following power series, find the radius of convergence R. Describe the set of all points where the power series converges absolutely, describe the set of all points where the power series converges conditionally, and describe the set of all points where the power series diverges.

- $\displaystyle\sum_{n=0}^{\infty} (x+5)^n$.
- $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{2^n\sqrt{n}}$.
- $\displaystyle\sum_{n=0}^{\infty} n!x^n$.
- $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

Exercise 3. For the following series, find the radius of convergence R. Describe the set of all points where the series converges absolutely, describe the set of all points where the series converges conditionally, and describe the set of all points where the series diverges. Finally, find the sum of the series as a function of x.

- $\displaystyle\sum_{n=0}^{\infty} (\ln x)^5$.
- $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^{2n}}{4n}$.
- $\displaystyle\sum_{n=0}^{\infty} n(n-1)x^{n-2}$.

Exercise 4. Find the Maclaurin series for the following functions. Find also the radius of convergence.

- $\dfrac{1}{1+x}$.
- $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$.
- $\dfrac{1 - \cos x}{x}$.

Exercise 5. Suppose f is a differentiable function such that $f^{(n)}$ exists and is continuous on the whole real line, for all positive integers n. Is it true that

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n \qquad ?$$

Explain your reasoning.