

Please provide complete and well-written solutions to the following exercises.

Due **November 25**, at the beginning of class.

The homework is due on WEDNESDAY this week!

Assignment 7

Exercise 1. Let $f: [0, 8] \rightarrow \mathbf{R}$ be a function such that $f(0) = 1$, $f(1) = 2$, $f(2) = 4$, $f(3) = 2$, $f(4) = 0$, $f(5) = 6$, $f(6) = 1$, $f(7) = 2$ and $f(8) = 0$. Using four equal-width rectangles, find the Riemann sums of f evaluated at the right endpoints, evaluated at the left endpoints, and evaluated at the midpoints.

Exercise 2. Evaluate

$$\lim_{n \rightarrow \infty} \frac{1^5 + 2^5 + \cdots + n^5}{n^6},$$

by showing that the limit is $\int_0^1 x^5 dx$.

Exercise 3. Let $f: \mathbf{R} \rightarrow \mathbf{R}$ be continuous with two continuous derivatives. Find all functions f such that $f''(x) = 20x^3 - 12x^2 + 6x$.

Exercise 4. Two baseballs are thrown upward from the edge of a cliff of height 432 feet. The first ball is thrown upward with a speed of 48 ft/s, and the other ball is thrown upward a second later with a speed of 24 ft/s. Do the baseballs ever pass each other before hitting the ground?

Exercise 5. Let $a < b$ and let m, M be constants. For a continuous function f , we know from Property (9) for integrals that if $m \leq f(x) \leq M$ for all $x \in [a, b]$, then

$$m(b - a) \leq \int_a^b f(x) dx \leq M(b - a).$$

Use this property to estimate $\int_0^2 (x^3 - 3x + 3) dx$.

Exercise 6. Let $f, g: \mathbf{R} \rightarrow \mathbf{R}$ be integrable functions. Suppose $\int_0^9 f(x) dx = 5$ and $\int_0^9 g(x) dx = 7$. Find $\int_0^9 (3f(x) + 2g(x)) dx$.

Exercise 7. Using the Fundamental Theorem of Calculus, evaluate $\int_{-2}^3 (x^2 - 3) dx$.

Exercise 8. Using the Fundamental Theorem of Calculus, evaluate $\int_3^5 (x^3 + x^{-2} + \cos(x)) dx$.