Please provide complete and well-written solutions to the following exercises.

No due date, but the quiz in Week 6 in the discussion section (on November 3rd or 5th) will be based upon this homework.

Assignment 5

Exercise 1. Let $f(x) = (1+x)^{15}$. Near $x = 0$, show that the linear approximation of f is given by $1 + 15x$, meaning $(1+x)^{15} \approx (1+15x)$ when x is near zero.

Exercise 2. Using linear approximation, find an approximate value of the cosine of 31 degrees. (Recall that one degree is $\pi/180$ radians.)

Exercise 3. (**A speeding ticket?**) Suppose I am driving in a car, and there are police cameras that are stationed at certain mile markers. The first camera spots my license plate at 10 AM. Five miles down the road, the second camera spots my license plate at 10 : 04 AM. If my speed exceeded 74 miles per hour at any particular point in time, I will automatically be issued a ticket in the mail. Will I be issued a ticket?

Exercise 4. Let $a, b > 0$. Find the maximum value of $f(x) = x^a(1-x)^b$ on the interval $[0, 1]$.

Exercise 5. Let $a, b > 0$. Find the area of the largest rectangle that can be inscribed in the ellipse $x^2/a^2 + y^2/b^2 = 1$. You may assume that the rectangle is aligned with the axes, and that its vertices touch the ellipse.

Exercise 6. Find the minimum and maximum values of $f(x) = 2\sqrt{x^2+1} - x$ on the interval $[0, 2]$

Exercise 7. It is the zombie apocalypse. You are in the forest, five miles from a straight road. If you traveled in a straight line directly towards the road from your current position (which is not a good idea), you would then have to walk 10 miles along the road to get to the safe house. You can travel at two miles per hour in the forest, and you can travel at four miles per hour on the road. What is the shortest amount of time that it will take to get to the safe house at the end of the road?

Exercise 8. Let $f(x) = x^5 - 5x^3/3 + 1$. Find the critical points of f, and find the intervals where f is increasing and decreasing. Apply the first derivative test to each critical point of f.

Exercise 9. Let $f(x) = x^4/4 - x^2/2$. Find the critical points of f, and find the intervals where f is increasing and decreasing. Apply the first derivative test to each critical point of f.