Graduate Statistics, 541A, Spring 2019, USC — Instructor: Steven Heilman

Name: ______________________ USC ID: ______________ Date: ___________

Signature: ______________________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 1

This exam contains 6 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your book or any calculator on this exam. You *can* use your homeworks. You are required to show your work on each problem on the exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a theorem or proposition from class or the notes or the book you must indicate this** and explain why the theorem may be applied. It is okay to just say, "by some theorem/proposition from class."
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Do not write in the table to the right. Good luck![a]

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. (10 points) Two people take turns throwing darts at a board. Person A goes first, and each of her throws has a probability of 1/4 of hitting the bullseye. Person B goes next, and each of her throws has a probability of 1/3 of hitting the bullseye. Then Person A goes, and so on. With what probability will Person A hit the bullseye before Person B does?

2. (10 points) Let $X\colon \Omega \to \mathbf{R}$ be a random variable. Prove:

$$\mathbf{E}(e^X) \geq e^{\mathbf{E}X}.$$

3. (10 points) Suppose you flip a fair coin 80 times. During each coin flip, this coin has probability 1/2 of landing heads, and probability 1/2 of landing tails.

   Let A be the event that you get more than 50 heads in total. Show that

$$\mathbf{P}(A) \leq \frac{1}{10}.$$

4. (10 points) Let X be a Gaussian random variable with mean $\mu \in \mathbf{R}$ and variance $\sigma^2 > 0$. Compute $\mathbf{E}X^2$, by differentiating the exponential family, where

$$f_w(x) := h(x) \exp\Big(\sum_{i=1}^{2} w_i t_i(x) - a(w)\Big) \qquad \forall\, x \in \mathbf{R}, \quad \forall\, w = (w_1, w_2).$$

$$a(w) = \log \int_{\mathbf{R}} h(x) \exp\Big(\sum_{i=1}^{2} w_i t_i(x)\Big) d\mu(x), \qquad \forall\, w = (w_1, w_2).$$

Recall that in this case,

$$t_1(x) := x, \qquad t_2(x) := x^2, \qquad w_1 := \frac{\mu}{\sigma^2}, \qquad w_2 := -\frac{1}{2\sigma^2},$$

$$a(w) := -\frac{w_1^2}{4w_2} - \frac{1}{2}\log(-2w_2), \qquad h(x) = (2\pi)^{-1/2}.$$

(Scratch paper)