

---

Please provide complete and well-written solutions to the following exercises.

Due February 16, 2PM PST, to be uploaded in blackboard as a single PDF document (in the Assignments tab).

## Homework 4

**Exercise 1.** The Wheel of Fortune involves the repeated spinning of a wheel with 72 possible stopping points. We assume that each time the wheel is spun, any stopping point is equally likely. Exactly one stopping point on the wheel rewards a contestant with \$1,000,000. Suppose the wheel is spun 24 times. Let  $X$  be the number of times that someone wins \$1,000,000. Using the Poisson Approximation the Binomial, estimate the following probabilities:  $\mathbf{P}(X = 0)$ ,  $\mathbf{P}(X = 1)$ ,  $\mathbf{P}(X = 2)$ . (Hint: consider the binomial distribution with  $p = 1/72$ .)

**Exercise 2.** Count the number of distinct ways in which you can arrange the letters of the words: CATTERPILLAR, and ARUGULA.

**Exercise 3.** Suppose  $X$  is a random variable such that:  $p_X(-3) = .1$ ,  $p_X(-2) = .2$ ,  $p_X(-1) = .15$ ,  $p_X(0) = .2$ ,  $p_X(3) = .1$ ,  $p_X(5) = .15$ ,  $p_X(6) = .05$ , and  $p_X(10) = .05$ .

Compute the probabilities of the following events

- $X > 3$
- $4 < X < 7$  or  $X > 9$
- $0 < X < 4$  or  $7 < X \leq 10$ .

**Exercise 4.** Suppose the probability of that you receive a prize in the mail is  $1/7000000$ . Show that you need to receive about 7000000 pieces of mail so that the probability that you receive at least one prize in the mail is about  $1 - 1/e$ . (Let  $A_i$  be the event that you receive a prize in the mail in the  $i^{\text{th}}$  piece of mail. You should assume that the events  $A_1, A_2, \dots$  are all independent.)

**Exercise 5.** Let  $\Omega = \{-3, -2, -1, 0, 1, 2, 3\}$ . Let  $\mathbf{P}$  be the uniform probability law on  $\Omega$ . Suppose  $X(\omega) = \omega$  for all  $\omega \in \Omega$ . Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  so that  $f(x) = x^2$  for any  $x \in \mathbf{R}$ . Compute the PMF of  $f(X)$ .

**Exercise 6.** Let  $X$  have a geometric distribution with parameter  $p$ . Let  $n$  be a positive integer. Let  $Y = X^4$  and let  $Z = \min(X, n)$ .

Find the PMF of  $Y$ . Then, find the PMF of  $Z$ .