Probability, 170B, Winter 2017, UCLA — Instructor: Steven Heilman

Name: ______________________ UCLA ID: ______________ Date: ____________

Signature: ______________________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 1

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam. You are required to show your work on each problem on the exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a theorem or proposition from class or the notes or the book you must indicate this** and explain why the theorem may be applied. It is okay to just say, "by some theorem/proposition from class."
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Do not write in the table to the right. Good luck![a]

Problem	Points	Score
1	8	
2	10	
3	10	
4	10	
5	10	
Total:	48	

1. Label the following statements as TRUE or FALSE. If the statement is true, **explain your reasoning**. If the statement is false, **provide a counterexample and explain your reasoning**.

   (a) (2 points) Let $A_1, A_2, \ldots$ be subsets of a sample space Ω. Let $\mathbf{P}$ denote a probability law on Ω. Then

   $$\sum_{n=1}^{\infty} \mathbf{P}(A_n) = \mathbf{P}\left(\cup_{n=1}^{\infty} A_n\right)$$

   TRUE FALSE (circle one)

   (b) (2 points) Let X be a continuous random variable. Let f_X be the density function of X. Then, for any $t \in \mathbf{R}$, $\frac{d}{dt}\mathbf{P}(X \leq t)$ exists, and

   $$\frac{d}{dt}\mathbf{P}(X \leq t) = f_X(t).$$

   TRUE FALSE (circle one)

   (c) (2 points) Let $A_1, A_2, \ldots$ be subsets of a sample space Ω. Let $\mathbf{P}$ denote a probability law on Ω. Then

   $$\lim_{n\to\infty} \mathbf{P}(A_n) = \mathbf{P}\left(\cup_{n=1}^{\infty} A_n\right)$$

   TRUE FALSE (circle one)

   (d) (2 points) $[0, 1] = \bigcap_{j=1}^{\infty} \left(-\frac{1}{j}, 1 + \frac{1}{j}\right)$.

   TRUE FALSE (circle one)

2. (10 points) Let X and Y be nonnegative random variables. Recall that we can define

$$\mathbf{E}X := \int_0^\infty \mathbf{P}(X > t)dt.$$

Assume that $X \leq Y$. Conclude that $\mathbf{E}X \leq \mathbf{E}Y$.

3. (10 points) Let X be a uniformly distributed random variable on $[-1, 1]$. Let $Y := X^4$. Find f_Y. (Here f_Y denotes the density function of Y.)

4. (10 points) Let X, Y be independent random variables. Suppose X has moment generating function

$$M_X(t) = 1 + t^4, \qquad \forall\, t \in \mathbf{R}.$$

Suppose Y has moment generating function

$$M_Y(t) = 1 + t^2, \qquad \forall\, t \in \mathbf{R}.$$

Compute $\mathbf{E}\Big[(X+Y)^2\Big]$.

5. (10 points) Let $0 < p < 1$. Suppose you have a biased coin which has a probability p of landings heads, and probability $1 - p$ of landing tails, each time it is flipped. Also, suppose you have a fair six-sided die (so each face of the cube has a distinct label from the set $\{1, 2, 3, 4, 5, 6\}$, and each time you roll the die, any face of the cube is rolled with equal probability.)

   Let N be the number of coin flips you need to do until the first head appears. Now, roll the fair die N times. Let S be the sum of the results of the N rolls of the die.

   Compute $\mathbf{E}S$.

(Scratch paper)