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Please provide complete and well-written solutions to the following exercises.

Due January 21, in the discussion section.

## Homework 2

**Exercise 1.** Two fair coins are flipped. It is given that at least one of the coins is heads. What is the probability that the first coin is heads? (A flipped fair coin has either heads with probability  $1/2$ , or tails with probability  $1/2$ . In the real world, a coin has a small probability of landing on its side, but we are ignoring this possibility!)

**Exercise 2** (The Monty Hall Problem). This Exercise demonstrates the sometimes counterintuitive nature of conditional probabilities.

You are a contestant on a game show. There are three doors labelled 1, 2 and 3. You and the host are aware that one door contains a prize, and the two other doors have no prize. The host knows where the prize is, but you do not. Each door is equally likely to contain a prize, i.e. each door has a  $1/3$  chance of containing the prize. In the first step of the game, you can select one of the three doors. Suppose the selected door is  $i \in \{1, 2, 3\}$ . Given your selection, the host now reveals one of the two remaining doors, demonstrating that this door contains no prize. The game now concludes with a choice. You can either keep your current door  $i$ , or you can switch to the other unopened door. You receive whatever is behind your selected door. The question is: should you switch or not?

If you do not switch your door choice, show that your probability of getting the prize at the end of the game is  $1/3$ . If you *do* switch your door choice, show that your probability of getting the prize is  $2/3$ . In conclusion, in this game, you should always switch your choice of doors.

**Exercise 3.** Suppose you roll three distinct fair, four-sided dice. What is the probability that the sum of the dice is 7?

**Exercise 4.** Two people take turns throwing darts at a board. Person  $A$  goes first, and each of her throws has a probability of  $1/4$  of hitting the bullseye. Person  $B$  goes next, and each of her throws has a probability of  $1/3$  of hitting the bullseye. Then Person  $A$  goes, and so on. With what probability will Person  $A$  hit the bullseye before Person  $B$  does?

**Exercise 5.** Suppose you roll two distinct fair six-sided dice. Suppose you roll these two dice again. What is the probability that both rolls have the same sum?

**Exercise 6.** Around 5% of men are colorblind, and around .25% of women are colorblind. Given that someone is colorblind, what is the probability that they are a man? (For the purpose of this problem, every person is either a man or a woman.)

**Exercise 7.** Two people are flipping fair coins. Let  $n$  be a positive integer. Person  $I$  flips  $n + 1$  coins. Person  $II$  flips  $n$  coins. Show that the following event has probability  $1/2$ : Person  $I$  has more heads than Person  $II$ .

**Exercise 8.** Suppose a test for a disease is 99.9% accurate. That is, if you have the disease, the test will be positive with 99.9% probability. And if you do not have the disease, the test will be negative with 99.9% probability. Suppose also the disease is fairly rare, so that roughly 1 in 20,000 people have the disease. If you test positive for the disease, with what probability do you actually have the disease?