Please provide complete and well-written solutions to the following exercises.

Due January 7th, in the discussion section.

(This Review Assignment will be collected but not be graded.)

Preliminary Review Assignment

Exercise 1. As needed, refresh your knowledge of proofs and logic by reading the following document by Michael Hutchings: http://math.berkeley.edu/~hutching/teach/proofs.pdf

Exercise 2. Take the following quizzes on logic and set theory:

http://scherk.pbworks.com/w/page/14864234/Quiz%3A%20Logic
http://scherk.pbworks.com/w/page/14864241/Quiz%3A%20Sets

(These quizzes are just for your own benefit; you don't need to record your answers anywhere.)

Exercise 3. Prove the following assertion by induction:

For any natural number n, $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$.

Exercise 4. Prove that the set of real numbers $\mathbf{R}$ can be written as the countable union

$$\mathbf{R} = \bigcup_{j=1}^{\infty}[-j, j].$$

(Hint: you should show that the left side contains the right side, and also show that the right side contains the left side.)

Prove that the singleton set $\{0\}$ can be written as

$$\{0\} = \bigcap_{j=1}^{\infty}[-1/j, 1/j].$$

Exercise 5. Let $\Omega = \{1, 2, 3, \ldots, 10\}$. Find sets $A_1, A_2, A_3 \subseteq \Omega$ such that: $A_1 \cap A_2 = \{2, 3\}$, $A_1 \cap A_3 = \{3, 4\}$, $A_2 \cap A_3 = \{3, 5\}$, $A_1 \cap A_2 \cap A_3 = \{3\}$, and such that $A_1 \cup A_2 \cup A_3 = \{2, 3, 4, 5\}$.