Probability, 170A, Spring 2016, UCLA — Instructor: Steven Heilman

Name: ______________ UCLA ID: ______________ Date: ______________

Signature: ______________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 1

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam. You are required to show your work on each problem on the exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a theorem or proposition from class or the notes or the book you must indicate this** and explain why the theorem may be applied. It is okay to just say, "by some theorem/proposition from class."
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Do not write in the table to the right. Good luck![a]

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

[a]May 16, 2016,

1. Label the following statements as TRUE or FALSE. If the statement is true, **explain your reasoning**. If the statement is false, **provide a counterexample and explain your reasoning**.

   (a) (2 points) The negation of the statement "For every integer j, $j^2 + j \geq 0$" is: "There exists an integer j such that $j^2 + j \leq 0$."

   TRUE FALSE (circle one)

   (b) (2 points) Let $A_1, A_2, \ldots$ be sets in a universe Ω. If $x \in \left(\cup_{j=1}^{\infty} A_j\right)^c$, then $x \in \cap_{j=1}^{\infty} A_j$.

   TRUE FALSE (circle one)

   (c) (2 points) Let A, B be subsets of a sample space Ω. Assume that $\mathbf{P}(B) > 0$. If $\mathbf{P}(A|B) = \mathbf{P}(B)$, then the sets A and B are independent.

   TRUE FALSE (circle one)

   (d) (2 points) Let $\Omega = \{1, 2, 3, 4, 5, 6, 7\}$. For any $A \subseteq \Omega$, define $\mathbf{P}(A)$ to be the number of elements in A. Then $\mathbf{P}$ is a probability law on Ω.

   TRUE FALSE (circle one)

   (e) (2 points) $[0, 1] = \bigcap_{j=1}^{\infty} \left(-\frac{1}{j}, 1 + \frac{1}{j}\right)$.

   TRUE FALSE (circle one)

2. (10 points) Prove the following assertion by induction on n:

For any positive integer n, we have $1 + 2 + 3 + \cdots + n = n(n+1)/2$.

3. (10 points) Suppose I have a bin with exactly 3 red, 4 green, 5 blue, and 6 yellow cubes. Step 1: I remove one cube from the bin uniformly at random, put the cube outside of the bin, then pause for one second. Step 2: I remove another cube from the bin uniformly at random, put the cube outside of the bin, then pause for one second. Step 3: I remove another cube from the bin uniformly at random, put the cube outside of the bin, then pause for one second. Let R be the event that a red cube is removed in Step 1. Let G be the event that a green cube is removed in Step 2. Let B be the event that a blue cube is removed in Step 3. Let A be the event $A = R \cap G \cap B$. What is $\mathbf{P}(A)$?

   (As usual, you must justify your answer; also you do not need to simplify your final answer.)

4. (10 points) Suppose a test for a disease is 98% accurate. That is, if you have the disease, the test will be positive with 98% probability. And if you do not have the disease, the test will be negative with 98% probability. Suppose also the disease is fairly rare, so that roughly 1 in $100,000$ people have the disease. If you test positive for the disease, with what probability do you actually have the disease? (Hint: let B be the event that you test positive for the disease. Let A be the event that you actually have the disease. Compute a conditional probability.)

5. (10 points) Two people are flipping fair coins. Let n be a positive integer. Person I flips $n+1$ coins. Person II flips n coins. Show that the following event has probability 1/2: Person I has more heads than Person II.

(Scratch paper)