Probability, 170A, Winter 2016, UCLA — Instructor: Steven Heilman

Name: ______________________ UCLA ID: ______________ Date: ____________

Signature: ______________________.
(By signing here, I certify that I have taken this test while refraining from cheating.)

Mid-Term 1

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this exam. You are required to show your work on each problem on the exam. The following rules apply:

- You have 50 minutes to complete the exam, starting at the beginning of class.
- **If you use a theorem or proposition from class or the notes or the book you must indicate this** and explain why the theorem may be applied. It is okay to just say, "by some theorem/proposition from class."
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this. Scratch paper appears at the end of the document.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do not write in the table to the right. Good luck![a]

1. Label the following statements as TRUE or FALSE. If the statement is true, explain your reasoning. If the statement is false, provide a counterexample and explain your reasoning.

   In the following statements, let A, B, C be subsets of some universe Ω. Let $\mathbf{P}$ denote a probability law on Ω. Also, let $A_1, A_2, A_3, \ldots$ be subsets of Ω.

   (a) (2 points) $\mathbf{P}(A \cup B) \leq \mathbf{P}(A) + \mathbf{P}(B)$.

   TRUE FALSE (circle one)

   (b) (2 points) $\mathbf{P}(A \cup B \cup C) = \mathbf{P}(A) + \mathbf{P}(A^c \cap B) + \mathbf{P}(A^c \cap B^c \cap C^c)$.

   TRUE FALSE (circle one)

   (c) (2 points) $\mathbf{P}\left(\bigcup_{k=1}^{\infty} A_k\right) = \sum_{k=1}^{\infty} \mathbf{P}(A_k)$.

   TRUE FALSE (circle one)

   (d) (2 points) If $\mathbf{P}(A) > 0$ and if $\mathbf{P}(A^c) > 0$, then $\mathbf{P}(B) = \mathbf{P}(B|A)\mathbf{P}(A) + \mathbf{P}(B|A^c)\mathbf{P}(A^c)$.

   TRUE FALSE (circle one)

   (e) (2 points) If $\mathbf{P}(A_1 \cap A_2 \cap A_3) = \mathbf{P}(A_1)\mathbf{P}(A_2)\mathbf{P}(A_3)$, then the sets A_1, A_2, A_3 are independent.

   TRUE FALSE (circle one)

2. (10 points) Let $\mathbf{R}$ denote the set of real numbers. Prove that

$$\mathbf{R} = \bigcup_{j=1}^{\infty} [-j, j].$$

3. (10 points) Two people take turns throwing darts at a board. Person A goes first, and each of her throws has a probability of $1/4$ of hitting the bullseye. Person B goes next, and each of her throws has a probability of $1/3$ of hitting the bullseye. Then Person A goes, and so on. With what probability will Person A hit the bullseye before Person B does?

4. (10 points) Suppose you have 100 cubes in a container, and they are labelled with the integers 1 through 100, so that each cube has a distinct integer label.

   Suppose you remove one cube uniformly at random from the container, and then put the cube back into the container. Then, you take one cube out of the container uniformly at random, and you then take another cube out of the container uniformly at random, you swap the integer labels of these two cubes, and you then put the cubes back into the container. Then, once again, you take one cube out of the container uniformly at random, and you then take another cube out of the container uniformly at random, you swap the integer labels of these two cubes, and you then put the cubes back into the container

   Finally, remove one cube from the container uniformly at random. What is the probability that this cube has the integer label 57?

5. (10 points) A community has $m > 0$ families. Each family has at least one child. The largest family has $k > 0$ children. For each $i \in \{1, \ldots, k\}$, there are n_i families with i children. So, $n_1 + \cdots + n_k = m$. Choose a child randomly in the following two ways.

   Method 1. First, choose one of the families uniformly at random among all of the families. Then, in the chosen family, choose one of the children uniformly at random.

   Method 2. Among all of the $n_1 + 2n_2 + 3n_3 + \cdots + kn_k$ children, choose one uniformly at random.

   What is the probability that the chosen child is the first-born child in their family, if you use Method 1?

   What is the probability that the chosen child is the first-born child in their family, if you use Method 2?

   (Your final answers MUST be expressed in terms of $m, n_1, \ldots, n_k$.)

(Scratch paper)