Please provide complete and well-written solutions to the following exercises.

Due January 5th or January 7th.

(This Review Assignment will not be collected. This Review Assignment will not be graded.)

Preliminary Review Assignment

Exercise 1. As needed, refresh your knowledge of proofs and logic by reading the following document by Michael Hutchings: http://math.berkeley.edu/∼hutching/teach/proofs.pdf

Exercise 2. Read the game rules for connect four, checkers, chess and go. We will be discussing these examples in class throughout the quarter.

Even if you are familiar with chess already, make sure to read the entire set of rules. There are many rules about stalemates that are a bit obscure, but they are important for this class, since these rules force the game to not last forever.

Exercise 3. Take the following quizzes on logic, set theory, and functions. (This material should be review from 115A.):

http://scherk.pbworks.com/w/page/14864234/Quiz%3A%20Logic
http://scherk.pbworks.com/w/page/14864241/Quiz%3A%20Sets
http://scherk.pbworks.com/w/page/14864227/Quiz%3A%20Functions

(These quizzes are just for your own benefit; you don't need to record your answers anywhere.)

Exercise 4. Prove the following assertion by induction:

For any natural number n, $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$.

Exercise 5. Write the following numbers in binary: $1, 3, 5, 8, 9$. Compute the following sums modulo 2: $1+3$, $4+9$, $10^{10}+1$.

Exercise 6. Tic-tac-toe is a partisan combinatorial game. Prove that it is progressively bounded. Then, try to figure out which of the following situations is true: the first player has a winning strategy; the second player has a winning strategy; both players have a strategy forcing at least a draw. (You do not need to give a formal proof of which situation holds, just try to guess the answer by playing a few tic-tac-toe games and by drawing on your personal experience.)