Please provide complete and well-written solutions to the following exercises.

Due September 27, in the discussion section.

(This Review Assignment will be collected, but this Review Assignment will not be graded.)

Preliminary Review Assignment

Exercise 1. As needed, refresh your knowledge of proofs and logic by reading the following document by Michael Hutchings: http://math.berkeley.edu/∼hutching/teach/proofs.pdf

Exercise 2. Take the following quizzes on logic, set theory, and functions. (This material should be review from 115A.):

http://scherk.pbworks.com/w/page/14864234/Quiz%3A%20Logic
http://scherk.pbworks.com/w/page/14864241/Quiz%3A%20Sets
http://scherk.pbworks.com/w/page/14864227/Quiz%3A%20Functions

(These quizzes are just for your own benefit; you don't need to record your answers anywhere.)

Exercise 3. Prove the following assertion by induction:

For any natural number n, $1^2 + 2^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$.

Exercise 4. Find a continuous function $f\colon \mathbf{R} \to \mathbf{R}$ such that f has a global maximum at $x = 0$, but f is not differentiable at 0.

Exercise 5. Let $f\colon [-1,2] \to \mathbf{R}$ be defined by $f(x) = x^3 - 3x + 2$. Find all local and global extrema of f.

Exercise 6. Find a continuous function $f\colon (0,1) \to \mathbf{R}$ such that there does not exist $x \in (0,1)$ such that $f(x) \leq f(z)$ for all $z \in (0,1)$.

Exercise 7. Find all eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 2 \end{pmatrix}$.

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Exercise 8. Prove that a real $n \times n$ matrix has at least one eigenvalue.