Please provide complete and well-written solutions to the following exercises.

No due date, but the quiz in Week 8 in the discussion section (on October 11) will be based upon this homework.

Q7: Quiz 7 Problems

Exercise 1. Let $f\colon [0,8]\to\mathbf{R}$ be a function such that $f(0)=1$, $f(1)=2$, $f(2)=4$, $f(3)=2$, $f(4)=0$, $f(5)=6$, $f(6)=1$, $f(7)=2$ and $f(8)=0$. Using four equal-width rectangles, find the Riemann sums of f evaluated at the right endpoints, evaluated at the left endpoints, and evaluated at the midpoints of the rectangles.

Exercise 2. Evaluate
$$\lim_{n\to\infty}\frac{1^5+2^5+\cdots+n^5}{n^6},$$
by showing that the limit is $\int_0^1 x^5dx$.

Exercise 3. Let $f\colon\mathbf{R}\to\mathbf{R}$ be continuous with two continuous derivatives. Find all functions f such that $f''(x)=20x^3-12x^2+6x$.

Exercise 4. Two baseballs are thrown upward from the edge of a cliff of height 432 feet. The first ball is thrown upward with a speed of 48 ft/s, and the other ball is thrown upward a second later with a speed of 24 ft/s. Do the baseballs ever pass each other before hitting the ground? (Acceleration due to gravity is assumed to be a constant -32 in these units.)

Exercise 5. Let $a<b$ and let m,M be constants. For a continuous function f, we know from Property (9) for integrals that if $m\leq f(x)\leq M$ for all $x\in[a,b]$, then
$$m(b-a)\leq\int_a^b f(x)dx\leq M(b-a).$$
Use this property to estimate $\int_0^2(x^3-3x+3)dx$.

Exercise 6. Let $f,g\colon\mathbf{R}\to\mathbf{R}$ be integrable functions. Suppose $\int_0^9 f(x)dx=5$ and $\int_0^9 g(x)dx=7$. Find $\int_0^9(3f(x)+2g(x))dx$.

Exercise 7. Using the Fundamental Theorem of Calculus, evaluate $\int_{-2}^3(x^2-3)dx$.

Exercise 8. Using the Fundamental Theorem of Calculus, evaluate $\int_3^5(x^3+x^{-2}+e^x)dx$.