Please provide complete and well-written solutions to the following exercises.

No due date, but the quiz in Week 7 in the discussion section (on October 4) will be based upon this homework.

Q6: Quiz 6 Problems

Exercise 1. (**A speeding ticket?**) Suppose I am driving in a car, and there are police cameras that are stationed at certain mile markers. The first camera spots my license plate at 10 AM. Five miles down the road, the second camera spots my license plate at $10:04$ AM. If my speed exceeded 74 miles per hour at any particular point in time, I will automatically be issued a ticket in the mail. Will I be issued a ticket?

Exercise 2. Let $f(x) = x^5 - 5x^3/3 + 1$. Find the critical points of f, and find the intervals where f is increasing and decreasing. Apply the first derivative test to each critical point of f.

Exercise 3. Let $f(x) = x^4/4 - x^2/2$. Find the critical points of f, and find the intervals where f is increasing and decreasing. Apply the first derivative test to each critical point of f.

Exercise 4. Find two numbers whose difference is 100 and whose product is a minimum.

Exercise 5. Suppose 1200 cm^2 of material is available to make a box with a square base and an open top. Find the largest possible volume of the box.

Exercise 6. Find the point on the line $y = 2x + 3$ that is closest to the origin.

Exercise 7. For a fish swimming at a speed v relative to the water, the energy expenditure per unit of time is proportional to v^3. It is believed that migrating fish try to minimize the total energy required to swim a fixed distance. If the fish are swimming against a current of speed u ($u < v$), then the time required to swim a distance L is $L/(v - u)$, and the total energy E required to swim the distance L is given by

$$E(v) = av^3 \frac{L}{v - u}$$

Here a is an arbitrary constant.

(a) Determine the value of v that minimizes E.
(b) Sketch the graph of E.

Note: This result has been verified experimentally. Migrating fish swim against a current at a speed 50% greater than the speed of the current.

Exercise 8. A cabinetmaker uses mahogany to produce 5 furnishings each day. Each delivery of one container of wood costs \$5000, and storage of that material is \$10 per day per unit stored, where a unit is the amount of material needed by her to produce 1 furnishing. How much material should be ordered each time and how often should the material be delivered to minimize her average daily cost in the production cycle between deliveries? (You can consider one container of wood to have an unlimited capacity, and the storage cost of one day is equal to the number of units of wood in the shop at the beginning of the day.)

Exercise 9. (**How we cough**) When we cough, the trachea (windpipe) contracts to increase the velocity of the air going out. This raises the question of how much it should contract to maximize the velocity of air, and whether the trachea really contracts that much when we cough.

Let r_0 be the rest radius of the trachea in centimeters, and let c be a positive constant whose value depends in part on the length of the trachea. Under reasonable assumptions about how the air near the wall is slowed by friction, the average air flow velocity v can be modeled by the equation

$$v = c(r_0 - r)r^2 \ \text{cm/sec}, \qquad \frac{r_0}{2} \leq r \leq r_0.$$

Show that v is greatest when $r = (2/3)r_0$. That is, the velocity is greatest when the trachea is about 33% contracted. The remarkable fact is that X-ray photographs confirm that the trachea contracts about this much during a cough.

Exercise 10. The average car achieves its best fuel efficiency at a speed of around 50 or 55 miles per hour. Driving faster than this speed can drastically reduce fuel efficiency, as we now show.

The fuel efficiency $f(v)$ of a car (in miles per gallon) traveling at a velocity v (in miles per hour) can be roughly modelled as

$$f(v) = (200,000)\frac{v}{v^3 + 2 \cdot 50^3}$$

When v is small, resistance from the tires, the mechanical aspects of the engine, etc. contribute to the inefficiency. When v is large, resistance from air also becomes a factor.

Find the maximum fuel efficiency of the car. Compare this efficiency to the values $v = 60$, $v = 70$ and $v = 80$. (Due to air friction and other frictional forces, driving faster is often less efficient.)