
Please provide complete and well-written solutions to the following exercises.

No due date, but the quiz in Week 4 in the discussion section (on September 13) will be based upon this homework.

Q4: Quiz 4 Problems

Exercise 1. One of the formulas for inventory management says that the average weekly cost of ordering, paying for, and holding merchandise is

$$A(q) = \frac{km}{q} + cm + \frac{hq}{2},$$

where q is the quantity you order when things run low (shoes, radios, brooms, or whatever the item might be); k is the cost of placing an order (the cost is the same, no matter how often you make an order); c is the cost of one item (a constant); m is the number of items sold each week (a constant); and h is the weekly holding cost per item (a constant that takes into account things such as space, utilities, insurance, and security). Find dA/dq and d^2A/dq^2 , and interpret your results in terms of the constants.

Exercise 2. Find the equation of the tangent to the curve at the given point.

(a) $y = 4(1 + x^3)(x + x^{10})$, $(x, y) = (1, 16)$, (b) $y = \frac{x^2 - 1}{x^2 + 1}$, $(x, y) = (0, -1)$.

Exercise 3. Find the equation of the tangent line to the curve at the given point.

(a) $y = \sqrt{1 + 4x^2}$, $(x, y) = (0, 1)$, (b) $x^2 + 2y^2 = 1$, $(x, y) = (1/\sqrt{2}, -1/2)$.

Exercise 4. Let $f(x) = x^{2/3}$. Draw $f(x)$ and $f'(x)$ on the same plot.

Exercise 5. Let $P(t)$ represent the price of a share of stock of a corporation at time t . What does each of the following statements tell us about the signs of the first and second derivatives of $P(t)$?

- “The price of the stock is rising faster and faster.”
- “The price of the stock is close to bottoming out”

Exercise 6. In economics, total utility refers to the total satisfaction from consuming some commodity. According to the economist Samuelson:

“As you consume more of the same good, the total (psychological) utility increases. However, . . . with successive new units of the good, your total utility will grow at a slower and slower rate because of a fundamental tendency for your psychological ability to appreciate more of the good to become less keen.”

- Sketch the total utility as a function of the number of units consumed.
- In terms of derivatives, what is Samuelson saying?

Remark 1 (An excerpt from wikipedia). U.S. President Richard Nixon, when campaigning for a second term in office announced that the rate of increase of inflation was decreasing, which has been noted as “the first time a sitting president used the third derivative to advance his case for re-election.” Since inflation is itself a derivative—the rate at which the purchasing power of money decreases—then the rate of increase of inflation is the derivative of inflation, or the second derivative of the function of purchasing power of money with respect to time. Stating that a function is decreasing is equivalent to stating that its derivative is negative, so Nixon’s statement is that the second derivative of inflation—or the third derivative of purchasing power—is negative.